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ABSTRACT

Earlier research by the authors in the design and use of computer-assisted instructional systems and curricula for teaching mathematical logic to gifted elementary school students has been extended to the teaching of university-level courses. This report is a description of the curriculum and problem types of a computer-based course offered at Stanford University, Introduction to Symbolic Logic. The data on which the report is based are from the spring and fall quarters of 1973, during which time 79 students enrolled in the course. The instructional program was written in LISP 1.5 for the DEC PDP-10 computer at the Stanford Institute for Mathematical Studies in the Social Sciences. Included in the report are examples of lesson routines, data on student effort and responses related to the course, and profiles of two students who took the course. (DGC)

COMPUTER-ASSISTED INSTRUCTION IN ELEMENTARY LOGIC AT THE UNIVERSITY LEVEL

by

Adele [illegible] and Patrick Suppes

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COMPUTER-ASSISTED INSTRUCTION IN ELEMENTARY LOGIC AT THE UNIVERSITY LEVEL[1]

Adele Goldberg and Patrick Suppes

Our earlier research in the design of instructional systems and curriculums for teaching mathematical logic to gifted elementary school students has been extended to the teaching of university-level courses (Goldberg, 1973; Suppes, 1972; Suppes & Ihrke, 1970). In this report, we describe the curriculum and problem types of a computer-based course offered at Stanford University: Philosophy 57A, Introduction to Symbolic Logic. We base our description on an analysis of the work of 79 students. Data on these students were collected during the third and fourth quarters (spring and fall, 1973) in which the course was offered. The instructional program was written in LISP 1.5 for the DEC PDP-10 at the Institute for Mathematical Studies in the Social Sciences (IMSSS). Programming details of the computer-based system, proof checker, and lesson driver are provided elsewhere (Goldberg, 1973, 1974).

Course Description

The main objective of the Stanford logic course is to familiarize the student with an exact and complete theory of logical inference. The course is taught solely by computer; all material is presented on the terminal and all problems are solved through interactions with a mechanical proof checker. Seminars, with optional attendance, were held several times during the spring quarter to discuss special topics. Attendence was low, so seminars were not held in the fall.

No book was required, although appropriate chapters of Introduction to Logic by Suppes (1957) were recommended. Depending on the class size, one or two teaching assistants, usually graduate students in the Philosophy Department, were available 9 hours each week to answer questions arising from the computer-based curriculum. A research programmer was also available.

An outline of the course is shown in Table I. Problems given the students emphasize proving arguments valid by constructing proofs in a

Insert Table I about here

natural deduction system (Lessons 401-408), or proving arguments invalid by either the method of truth analysis (Lessons 403 and 409) or of interpretation (Lessons 423 and 428). The method of interpretation is also applied to prove premises consistent, or axioms of a theory independent (Lesson 429).

Beginning with Lesson 415, examples of axiomatically formulated theories are introduced. Two examples, the elementary theory of Abelian groups and the elementary theory of non-Abelian groups, are given in Lessons 415 through 420. The axioms and theorems studied in these lessons are listed in Table II. Numerous other examples, in the form of

Insert Table II about here

finding-axioms exercises, range from the algebra of real numbers to a segment of elementary geometry.

Lesson 421 teaches the student how to do the finding-axioms exercises: how to specify a set of axioms from a given list of statements, and how to

TABLE 1

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Outline of Lessons 401-435

Lesson	Number of problems	Average number of hours	Content
401	19	.31	Well-formed expressions of propositional logic
402	16	.21	Semantics for propositional logic--truth tables
403	18	.50	Truth analysis Definition of a derivation and use of modus ponens (AA)
404	22	1.29	Conditional proof procedure (CP) Working premise (WP) Role of subsidiary derivations
405	14	3.12	Practice with the conditional proof procedure New commands: Delete last line(s) (DLL) Hypothesis (HYP) Definition of a theorem
406	24	1.56	Inference rules: Double negation (DN) Form a conjunction (FC) Right conjunct (RC) Left conjunct (LC)
407	21	1.00	Inference rules: Form a disjunction (FD) Deny a disjunct (DD) Deny the consequent (DC)
408	28	4.43	Indirect proof procedure (IP) Proof of De Morgan's Law Inference rules: De Morgan's Law (DM) Commute the disjunction (CD) Commute the conjunction (CC)
409	26	2.57	Constructing counterexamples by truth assignment Problems require student to decide whether argument valid--construct derivation ([illegible])--or invalid--construct counter-example (CEX)

(Table I, cont.)

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Lesson	Number of problems	Average number of hours	Content
409 (cont.)			Inference rule: Definition of implication symbol (DFA)
410	29	.42	Elementary algebra: Well-formed formulas using equality (=) and inequality (>,<) relations Number definition (ND)
411	28	.44	Rules about equality: Commute equality (CE) Add equal term (AE)
412	30	.53	Rules about equality Subtract equal term (SE) Logical truth (LT)
413	23	.23	Review Replace equality (RE)
414	20	.87	More practice with RE Introduction to the INIT command to make up one's own problems
415	32	1.04	Definition of "axiom" and "instance of an axiom" First axiom for a commutative group: Commute addition (CA)
416	23	1.02	Second axiom for a commutative group: Associative law (AS) Short forms of AS: Associate left (AL) Associate right (AR)
417	29	1.10	Remaining axioms for a commutative group: Zero (Z) Negative number (N) Additive inverse (AI)
418	30	1.44	Theorems on addition Theorems 1-3 Using theorems in a derivation Short forms for theorems and axioms
419	19	1.13	Theorems 5-7
420	11	1.54	Axioms and theorems for a noncommutative group Reprove theorems 1-7 without commutative axiom

(Table I, cont.) **BEST COPY AVAILABLE**

Lesson	Number of problems	Average number of hours	Content
421	14	.40	Explanation of the finding-axioms exercises Exercises 1-5
422	39	1.55	Translating English sentences into first-order logic (no quantifiers)
423	26	.48	Interpretation of a sentence: concept of a valid argument How to provide interpretations in an algebraic domain
424	21	1.64	Counterexamples by interpretation
425	53	1.21	Introduction to universal and existential quantification Translating English sentences into first-order logic using quantifiers
426	44	2.66	Inference rules for quantifiers: Universal specification (US) Universal generalization (UG) Existential specification (ES) Existential generalization (EG)
427	49	1.74	Justification of restrictions on rules governing inferences with quantifiers Quantifier negation rules: QNA, QNB, QNC, QND
428	26	5.05	Inference rules Commute equivalence (CE) Replace equivalence (RQ) Extension of theory of interpretation to sentences having quantifiers Problems in which student decides on validity of an argument in English
429	24	5.96	Consistency of premises Independence of axioms (applications of theory of interpretation)
431	34	1.63	Boolean algebra--axioms Commute union (CU) Commute intersection (CI) Distribute union (DU) Distribute intersection (DI) Intersection identity (II) [illegible] (A) Law of excluded middle (EM)

(Table I, cont.)

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Lesson	Number of problems	Average number of hours	Content
432	50	5.36	Boolean algebra Duality Theorems 161-182
433	16	3.25	Boolean algebra Axioms Subclass (SA) Theorems 183-192
435	48	4.12	Symbolization of English sentences especially related to the predicate calculus with identity; proof of equivalence of forms

TABLE II BEST COPY AVAILABLE

Axiom and Theorem List for Lessons 401-429

Rules of Inference

CON (P → Q) IFF (NOT Q → NOT P)
DFA (P → Q) IFF (NOT P OR Q)
DNA NOT (P → Q) IFF (P & NOT Q)

FOR ANY FORMULA S,
QNA (A X) S(X) IFF NOT (E X) NOT S(X)
QNB (A X) NOT S(X) IFF NOT (E X) S(X)
QNC (E X) S(X) IFF NOT (A X) NOT S(X)
QND (E X) NOT S(X) IFF NOT (A X) S(X)

For the following axioms and theorems, assume universal quantification unless otherwise specified.

Axioms on Addition

CA (commutativity):	X + Y = Y + X
AS (associativity):	(X + Y) + Z = X + (Y + Z)
Z (zero axiom):	X + 0 = X
N (negative number):	X + (-Y) = X - Y
AI (additive inverse):	X + (-X) = 0
U (unity axiom):	NOT 1 = 0

Theorems on Addition

TH1:	(-X) + X = 0
TH2:	0 + X = X
TH3:	X - X = 0
TH4:	0 - X = -X
TH5:	0 = -0
TH6:	X - 0 = X
TH7:	X + Y = X + Z → Y = Z
TH8:	X + Y = Z → X = Z - Y
TH9:	X = Z - Y → X + Y = Z
TH10:	X + Y = 0 → X = -Y
TH11:	X = -Y → X + Y = 0
TH12:	X + Y = X → Y = 0
TH13:	-(-X) = X
TH14:	(-(X + Y)) + Y = -X
TH15:	-(X + Y) = (-X) - Y

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(Theorems on Addition, cont.)

TH16: $(-X) - Y = (-Y) - X$
TH17: $-(X - Y) = Y - X$
TH18: $(X - Y) - Z = X + ((-Y) - Z)$
TH19: $(X - Y) - Z = X - (Y + Z)$
TH20: $X + (Y - X) = Y$
TH21: $X - (X + Y) = -Y$
TH22: $(X - Y) + (Y - Z) = X - Z$

Axioms on Order

AS (asymmetry):	$X < Y \rightarrow \text{NOT } Y < X$
AD (addition):	$X < Y \rightarrow X + Z < Y + Z$
TR (transitivity):	$X < Y \ \& \ Y < Z \rightarrow X < Z$
CN (connectivity):	$\text{NOT } X = Y \rightarrow X < Y \text{ OR } Y < X$
DG (definition >):	$X > Y \text{ IFF } Y < X$
NL (no least number):	$(A\ X)(E\ Y)\ Y < X$
NG (no greatest number):	$(A\ X)(E\ Y)\ X < Y$

Theorems on Order

TH61: $\text{NOT } X < X$
TH62: $X < Y \rightarrow \text{NOT } X = Y \ \& \ \text{NOT } Y < X$
TH63: $X < 0 \rightarrow 0 < -X$
TH64: $0 < -X \rightarrow X < 0$
TH65: $X + Y < X + Z \rightarrow Y < Z$
TH66: $X < Y \rightarrow -Y < -X$
TH67: $-Y < -X \rightarrow X < Y$
TH68: $X + (-Y) < X + (-Z) \rightarrow Z < Y$
TH69: $Z < Y \rightarrow X + (-Y) < X + (-Z)$

A Useful Relation

TH70: $(A\ X)\ X < X + 5$
TH71: $(A\ X)(A\ Y)\ X < Y + 5 \text{ OR } Y < X + 5$
TH72: $(E\ X)(E\ Y)(E\ Z)\ X < Y + 5 \ \& \ Y < Z + 5 \ \& \ \text{NOT } X < Z + 5$

Boolean Algebra Axioms and Theorems

CU $(G \vee H) = (H \vee G)$
UI $(G \vee 0) = G$
CI $(G \uparrow H) = (H \uparrow G)$
DU $(G \vee (H \uparrow K)) = ((G \vee H) \uparrow (G \vee K))$

(Boolean Algebra Axioms and Theorems, cont.)

DI [illegible] ↑ [illegible] ∨ [illegible] ↑ [illegible] ∨ [illegible] ↑ [illegible]
IT [illegible] ↑ [illegible]
RA (G ↑ (−[illegible]) [illegible] O)
FM [illegible] ∨ (−[illegible]) [illegible]
CA ((G ∨ H) ∨ K) [illegible] ∨ (H ∨ K))
IT [illegible] ∨ H) ∨ [illegible] ∨ (H ∨ K))
CL [illegible] ∨ (H ∨ K) [illegible] ((G ∨ H) ∨ K)
LA ((G ↑ H) ↑ K) = (G ↑ (H ↑ K))
TA [illegible] ↑ [illegible] INC [illegible]
[illegible] [illegible] INC H) [illegible] − [illegible] O)

TH161. [illegible] ∨ [illegible] (G ∨ H)
TH162 [illegible] ∨ [illegible] H)
TH163. [illegible] ∨ [illegible]
TH164 [illegible]
TH165 (G ∨ [illegible]
TH166 (G ↑ [illegible]
TH167: [illegible] ∨ [illegible] H)) [illegible]
TH168: [illegible] ↑ [illegible] ∨ [illegible]
TH169. (([illegible] ↑ [illegible]) & ((G ↑ H) = O)) → (G = O)
TH170. (([illegible] ∨ (−H)) [illegible] & ((G ∨ H) = U)) → (G = U)
TH171 [illegible] ∨ [illegible] → (G = [illegible])
TH172 [illegible]
TH173 [illegible] ∨ H) [illegible] ∨ K)) & ((G ↑ H) = (G ↑ K))) → (H = K)
TH174. ((([illegible] ∨ [illegible]) & ((G ∨ K) = U)) & (((G ↑ H) = O)
& ([illegible] K) = O))) → (H = K)
TH175. (((G ∨ [illegible]) & ((G ∨ K) = G)) & (((G ↑ H) = O)
& ([illegible] ↑ [illegible])) → (H = K)
TH176. ((((G ∨ H) [illegible] U) & ((G ∨ K) = U)) & (((G ↑ H) = G)
& (([illegible] K) = G))) → (H = K)
TH177 −(−G) [illegible]
TH178 −U = [illegible]
TH179. [illegible] ∨ H) [illegible] (G ↑ H)) → (G = H)
TH180. −(G ∨ [illegible]) = ((−G) ↑ (−H))
TH181 −(G ↑ H) = ((−G) ∨ (−H))
TH182: (G ↑ (H ↑ K)) = ((G ↑ H) ↑ (G ↑ K))
TH183 G INC G
TH184: O INC G
TH185: G INC U
TH186: ((G INC H) & (H INC G)) → (G = H)
TH187. (G INC H) → ((G ∨ H) = H)
TH188. ((G ∨ H) = H) → (G INC H)
TH189. ((G ∨ (−H)) = U) → (H INC G)
TH190. (H INC G) → ((G ∨ (−H)) = U)
TH191. [illegible] INC [illegible] H) [illegible]
TH192. ((G ↑ H) = G) → (G INC H)
TH193. ((G INC H) & (H INC K)) → (G INC K)

prove that the rest of the statements are theorems derivable from the axioms selected by use of the rules of logical inference so far introduced. The program itself determines whether the student has satisfactorily completed an exercise, providing a complete report on the axioms selected and on which axioms and lemmas were needed by the student to prove a given theorem.

Two problem formats are used in the lessons on translating English into the formalism of the first-order predicate calculus. Lesson 422 is restricted to iterative requests for possible translations until the student's response matches an instance of one of several stored correct answers. By Lesson 435, the student is expected to show a proficiency in determining whether or not two expressions are logically equivalent. Thus, if the student's symbolization of an English sentence does not correspond with one of those stored with the curriculum, he or she must decide whether it is possible to prove logical equivalence. The student uses his skills in constructing proofs to show that an if-and-only-if relationship holds, or uses the method of interpretation to show that the equivalence does not hold.

The system of inference for first-order predicate logic follows that of Suppes (1957). The problems in Lesson 426 motivate the restrictions on the use of the quantifier rules, and those in Lesson 427 have the student prove each of four quantifier negation rules. A large number of exercises at the end of each lesson give the students practice with the general principles introduced.

The student receives a passing grade in the course if he completes Lessons 401 through 429, and does the first five finding-axioms exercises.

Our intention was to make available a set of optional lessons from which the students could select ones to use to qualify for a grade of B. During the spring and fall quarters, only one such set of lessons was available: Lessons 431 through 433 on an axiomatization of Boolean algebra. Theorems for these lessons are also shown in Table II. Students desiring a grade of B did two additional finding-axioms exercises. Lesson 435, on symbolizing sentences in English, completed the course and the student's requirement for a grade of A.

The course curriculum is thus a linear sequence of lessons which the student follows. He can interrupt that sequence to make up his own problems, prove lemmas to help in proving problems presented to him, or move around in the course in a nonlinear fashion. This last feature proved beneficial when unplanned computer down-time meant that the student's history of problems completed was not properly recorded by the program. The student could skip ahead to the next problem in the sequence without waiting for a proctor to patch his history file. On the average, the students used this program feature 11 times (12 times in the fall) out of an average of 62 sessions (37 in the fall) at the terminal. We found that this simple feature improved the students' attitude towards studying with an instructional system sometimes prone to electronic error.

System Usage

A total of 179 students enrolled in the Stanford course during the four quarters from fall, 1972, to fall, 1973, with 121 students completing a grade of A, B, or pass. The distribution of students by quarter and

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grade received ... analysis ...

Insert Table III about here

this report are based on 79 students, 41 from the spring-1973 class, and 38 from the fall-1973 class.

Contrary to the regular pace of daily lectures, students' work habits resembled more the sporadic efforts considered characteristic of most research workers. Almost all of them abstained from any contact with the course for at least a week, possibly during periods when pressures in other courses mounted. During the spring term, 11 students took a break of at least one month.

As can be seen from Table III, the dropout rate was approximately 14.5 percent, with a slight bias upward in this number because students in the fall-1973 class did not have any period except the Christmas vacation to make up incompletes. University regulations allow them to finish the course within one year after enrollment. It is interesting to compare this rate with two other courses. The elementary introduction to philosophy, which is a general course not oriented toward logic at all, has a larger average enrollment than the logic course, and over the past 6 years the average dropout rate has been 13.6 percent. A course of a different sort, the first intermediate course in logic, which is technically harder and requires some mathematical and logical sophistication on the part of the students, has a smaller enrollment than the logic course by a factor of 3 or 4. Summing the enrollment over the past 6 years, the average dropout rate in that course has been 27.8 percent. It thus can be seen that the dropout rate is more or less comparable to other courses. It should be emphasized that the University has a system

TABLE III

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Distribution of Grades

Quarter	Grades at end of each quarter					
	A	B	Pass	No credit	Incom-plete	Total
Fall, 1972	13	3	9	14	13	52
Winter, 1972	3	0	4	2	12	21
Spring, 1973	20	0	4	4	18	46
Fall, 1973	22	0	11	12	15	60
Total	58	3	28	32	58	179

Quarter	Grades of incompletes finished as of January 1, 1974[a]				
	A	B	Pass	No credit	Total
Fall, 1972	2	4	3	4	13
Winter, 1972	3	1	4	4	12
Spring, 1973	3	3	4	8[b]	18
Fall, 1973	2	0	3	10[b]	5
Total	10	8	14	26	48

[a]According to University rules, students have one year after enrollment to complete a course.

[b]These students actually had further time available to complete the course.

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of enrollment [illegible]
courses and then [illegible] base of enrollment, once they have made a choice of what they want to take.

The average amount of actual time spent at a computer terminal (43.84 hours), combined with our estimate of time spent off the terminal working on harder problems or on the finding-axioms exercises (an estimated upper limit of 4[illegible] hours), is commensurate with the five units of academic credit the students received. Table IV gives the mean and

Insert Table IV about here

variance of time spent on all lessons. This timing information is not normalized by the number of problems nor the problem types, but does indicate the relative difficulty in learning certain proof procedures or concepts taught in each lesson. Thus we see time increases in lessons in which the students are taught indirect proof procedure and inference rules for quantifiers. The large variance in Lesson 435 highlights the character of the lesson. Either a student can easily translate the statements expressed in English into the predicate calculus, or he has to spend a significantly longer time trying different translations or proving that his translations are or are not logically equivalent to the stored answers.

Another view of system usage was compiled in order to examine each student's deviation from the average amount of time per lesson. Except for 9 students (8 fast, 1 slow), students were not uniformly fast or slow with respect to the average amount of time per lesson. Moreover,

TABLE IV

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Average Time per Lesson

Lesson	Average time in hours	Standard deviation	No. of students[a] completing lesson
401	.31	.21	42
402	.21	.05	42
403	.50	.37	43
404	1.29	.77	43
405	3.12	1.40	43
406	1.56	2.28	43
407	1.00	1.22	43
408	4.43	2.79	43
409	2.57	.95	43
410	.42	.20	43
411	.44	.24	43
412	.53	.28	43
413	.23	.07	42
414	.87	.63	43
415	1.04	.62	43
416	1.02	.47	43
417	1.10	.54	43
418	1.44	.84	43
419	1.13	.60	43
420	1.54	.93	43
421	.40	.20	43
422	1.55	.71	39
423	.48	.19	40
424	1.65	.61	37
425	1.61	.60	37
426	2.66	1.31	38
427	2.74	1.01	36
428	5.05	2.78	35

(Table IV, cont.)

Lesson	Average time in hours	Standard deviation	No. of students[a] completing lesson
429	5.96	3.30	32
431	1.63	.76	28
432	5.36	1.82	25
433	3.25	2.71	25
435	4.12	3.45	20

[a]Spring class only.

the students do not appear to slow down or speed up in a uniform manner, but are quite diverse in their deviation from the average.

Summaries of computer usage, total number of sessions each student spent at the terminal, number of minutes spent, and the number of central processor minutes each student's work required, are shown in Table V.

Insert Table V about here

Because the instructional system is basically a proof checker, considerably more computation time is required to carry out the computer-student interaction than is required by a traditional drill-and-practice system.

Students worked on Teletype (R) Model-33 terminals; terminals were available in a classroom at IMSSS. During the spring quarter, the students had access to 10 terminals from 3:00 p.m. until 6:00 a.m., Monday through Friday, and all day Saturday and Sunday. In the fall, the daily schedule was extended to allow all-day access during the week; Friday nights and Saturdays until 4:00 p.m. were reserved for machine maintenance by the IMSSS staff.

A summary of the actual times the students used the system is shown in terms of the total number of terminal hours per hour of the day in Table VI. Thus, the entry for hour 0 is the total number of terminal

Insert Table VI about here

hours logged between midnight and 1:00 a.m. In the spring, 9:00 p.m. was the most popular hour for working; while 2:00 p.m. was preferred by the fall students, it was not available to students in the spring. In each case, terminals were mainly utilized during hours when there

TABLE [illegible]

Number of Sessions, Connect Time, and CPU Cycles

for Each Student, Spring, 1973

Student identif. number	Number of sessions	Minutes of connect time	CPU minutes	Student identif. number	Number of sessions	Minutes of connect time	CPU minutes
2850	96	2491.65	122.65	2875	56	3603.32	110.33
2851	45	2019.[illegible]2	74.63	2877	37	1685.80	70.00
2852	57	3308.72	87.52	2878	33	1837.92	50.18
2853	52	3944.87	105.87	2879	17	2049.12	74.27
2854	24	1336.97	41.43	2880	44	3833.43	106.87
2855	90	3657.23	123.92	2881	29	1929.15	48.78
2856	20	2056.27	96.58	2882	73	3708.98	110.32
2857	20	4390.77	65.62	2883	91	3410.52	121.28
2858	60	2777.65	100.78	2884	18	1852.72	29.53
2859	58	2741.72	92.57	2886	26	2072.28	76.85
2860	31	4035.12	41.58	2888	17	1546.65	41.93
2861	49	2381.32	94.23	2889	67	2758.68	107.28
2863	32	2248.00	76.15	2890	35	3285.87	118.45
2864	65	3747.35	84.78	2891	69	2859.57	92.97
2865	66	3619.68	105.10	2892	13	807.95	32.77
2867	88	2661.62	111.98	2893	288	6710.22	200.73
2868	17	893.38	24.78	2894	218	4456.53	144.55
2869	37	1512.75	64.77	2896	27	1700.72	53.87
2870	59	2707.72	87.67	2897	40	1940.17	62.70
2871	59	3018.18	113.73	2898	40	2811.85	69.42
2872	57	1896.15	56.67	Averages	56.4	2742.52	85.45
2873	47	2877.75	92.70				

TABLE Vb

Number of Sessions, Connect Time, and CPU Cycles

for Each Student, Fall, 1973

Student identif. number	Number of sessions	Minutes of connect time	CPU minutes	Student identif. number	Number of sessions	Minutes of connect time	CPU minutes
1301	27	2680.97	75.55	1333	36	2079.32	64.42
1302	44	2357.02	67.17	1334	25	2180.47	54.05
1303	52	3791.35	80.40	1335	47	3297.02	95.28
1304	41	3518.07	38.52	1336	37	2474.85	63.63
1305	38	3425.22	81.67	1338	45	1977.37	53.80
1310	28	2070.32	61.42	1339	32	2764.12	95.07
1311	39	2738.05	90.68	1342	44	2458.50	69.83
1312	52	3013.20	73.48	1345	47	3766.53	94.40
1313	30	3877.23	225.28	1346	21	1798.17	60.18
1314	33	1853.22	40.62	1347	36	2647.22	82.98
1316	56	3453.50	67.23	1348	27	2220.40	44.15
1320	31	1766.35	41.75	1349	21	1331.65	53.23
1322	36	1710.37	71.38	1352	34	1821.97	39.33
1323	45	3333.15	130.68	1355	15	1216.63	34.87
1324	54	1857.67	62.08	1356	35	2824.07	72.87
1325	42	2274.87	63.35	1358	31	1436.17	44.67
1328	50	2357.65	85.25	1359	36	1748.63	57.78
1330	25	2009.05	59.28	1360	31	1570.50	63.05
1331	37	1853.17	61.90	Averages	36.9	2422.64	71.88
1332	43	2506.30	60.08				

[illegible]

Total Number of Hours Terminals Used

at Each Time of Day (Spring, [illegible])

Hours of the day	Total number of terminal hours
0	282
1	164
2	55
3	32
4	17
5	15
6	10
7	8
8	8
9	9
10	16
11	24
12	37
13	49
14	91
15	162
16	361
17	418
18	350
19	470
20	500
21	528
22	480
23	414

TABLE VIb

Total Number of Hours Terminals Used

at Each Time of Day (Fall, 1973)

Hours of the day	Total number of terminal hours
0	62
1	41
2	18
3	11
4	11
5	9
6	9
7	7
8	15
9	96
10	208
11	209
12	238
13	354
14	392
15	389
16	339
17	222
18	112
19	211
20	254
21	250
22	175
23	104

was a [illegible] the

popularity of the course in the self-paced work schedule.

Student Evaluation of the Course

To determine the students' own evaluation of the problems, we asked those students who completed Lesson 4[illegible]9 to rank order all problem types according to their preference in doing the type of problem, with the exception of the symbolization problems found in Lesson 435. Table VII

Insert Table VII about here

shows that derivations using the rules of conditional proof (CP) and indirect proof (IP) were preferred (also the most frequently encountered), while sentential derivations in general ranked second. The low rating of problems involving proof that axioms are independent or not, or premises inconsistent or not, is almost surely due to the low frequency of occurrence of these problem types in the curriculum. Data on the students' ability to make the required choices in these kinds of problems show that the students' intuitions for determining consistency of premises or independence of axioms were not as well formed as for determining the validity of an argument.

We also asked all the students to complete an attitude survey. Questions are shown in Appendix I; ratings on a 1-7 preference scale are given in Table VIII. For the most part, students enjoyed the course,

Insert Table VIII about here

would like to take other computer-based courses, and found they liked the active interaction the system afforded. They were not happy if a

TABLE VII

Student Ranking of Preferred Problem Types

Problem type	Choice									
	1	2	3	4	5	6	7	8	9	10
Multiple choice	4	2	5	12	3	0	0	3	3	5
Sentential derivation	3	7	7	4	5	2	3	2	1	3
Counterexample by truth assignment	6	4	5	6	5	4	0	2	4	1
Derivations using CP and IP	11	5	5	6	3	1	3	1	1	1
Proofs in elementary algebra	4	5	6	2	4	4	4	5	2	1
Finding axioms	4	4	1	4	3	3	6	5	2	5
Derivations of formulas having quantifiers	0	5	1	1	6	8	10	4	2	0
Counterexample by interpretation	1	2	7	1	5	4	7	5	4	1
Proving premises consistent or inconsistent	2	3	0	1	1	4	2	7	12	5
Proving axioms independent or dependent	2	1	0	0	2	7	3	2	5	15

TABLE VIII

 Results of Questionnaire (24 Students)

Question number	Scale						
	1	2	3	4	5	6	7
1	7	10	1	1	1	1	3
2	19	3	1	0	0	1	0
3	0	0	1	2	4	5	12
4	0	0	1	1	2	2	18
5	0	0	2	0	2	4	16
6	3	3	3	4	3	5	3
7	0	3	4	2	5	4	6
8	0	0	2	6	8	4	4
9	7	5	5	1	2	2	2
10	3	3	7.5	0	5	5.5	0
11	11	3	2	5	2	0	1
12	0	2	5	6	6	5	0
13	3	6	2	2	4	4	3
14	7	6	6	0	1	2	2
15	3	6	6	4	2	2	1

human tutor was not available. The spring students were displeased that the terminals were not available before 5:00 p.m. By an outstanding majority, the students liked working at their own pace at the terminals and enjoyed working on their own, independent of the activities of their classmates.

Based on comments made by the students on these questionnaires, several commands were added to the system in order to facilitate reviewing partial proofs and shortening solutions. Comments by students in the winter-1973 class precipitated the addition of the LESSON command for skipping around curriculum problems.

Example Problems

The examples we have chosen to describe in detail were selected for one or more of the following reasons: (1) the distribution of number of steps to complete a solution indicated a great variety of solutions, (2) rules required to solve the problem had a high percentage of errors, and (3) average number of hints requested for the problem showed that the problem was considered difficult by many students. In each example presentation, we provide a sample solution and the overall average, minimum, and maximum number of steps. Many problems are randomly selected. If a student stops a session in the middle of constructing a derivation, there is a good chance that he will not receive that same problem at his next session. Randomly assigned problems are numbered as: lesson.problem.1 or lesson.problem.2.

A table of the minimum and maximum number of solution steps for all derivation problems (600 such problems occur in the curriculum) is provided

in Appendix [illegible] and fall data.

In the initial lessons of the course, students learn to construct propositional or sentential derivations. They are taught a simple command language consisting of mnemonics for inference rules, and conditional and indirect proof procedures. The summary of inference rules given to the students appears in Appendix XII.

The first example, shown in Figure 1, is interesting because it requires a subsidiary derivation whose hypothesis is identical to one needed

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Insert Figure 1 about here

---

in the main derivation. At the point of presentation of this problem, the student has learned the rules modus ponens (AA), working premise (WP), and conditional proof procedure (CP).

Indirect proof. The indirect proof procedure (IP) is taught in Lesson 408, where the eighth problem, shown in Figure 2, is the first

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Insert Figure 2 about here

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derivation example. To encourage the use of IP, the inference rule denying the consequent of a conditional sentence is not permitted in solutions to any problems in this lesson. Control of this nature is part of the basic lesson driver. The tenth problem in the lesson, shown in Figure 3,

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Insert Figure 3 about here

---

is significant because three hints were available, and, on the average, students requested one hint. This was one of only ten problems in which

DERIVE (((R & Q) → Q) → ((R & Q) → S)) → ((R & Q) → (Q → S))

WP	(1)	((R & Q) → Q) → ((R & Q) → S)
WP	(2)	R & Q
WP	(3)	Q
WP	(4)	R & Q
4.3CP	(5)	(R & Q) → Q
1.5AA	(6)	(R & Q) → S
6.2AA	(7)	S
3.7CP	(8)	Q & S
2.8CP	(9)	(R & Q) → (Q & S)
1.9CP	(10)	((R & Q) → Q) → ((R & Q) → S)) → ((R & Q) → (Q → S))

Average number of steps: 10.3
Minimum number of steps: 10
Maximum number of steps: 15

Fig. 1. Sample derivation of Problem 405.12.2

DERIVE (NOT S) → (S → NOT Q)

WP	(1)	NOT S
WP	(2)	S
WP	(3)	Q
1.2.3IP	(4)	NOT Q
2.4CP	(5)	S → NOT Q
1.5CP	(6)	(NOT S) → (S → NOT Q)

Average number of steps: 6.2
Minimum number of steps: 6
Maximum number of steps: 10

Fig. 2. Sample derivation of Problem 408.8.2

DERIVE (NOT((W & R) → S)) → (W & R)

HYP	(1)	NOT ((W & R) → S)
WP	(2)	NOT (W & R)
WP	(3)	W & R
WP	(4)	NOT S
2.3.4IP	(5)	S
3.5CP	(6)	(W & R) → S
1.6.2IP	(7)	W & R
1.7CP	(8)	(NOT ((W & R) → S)) → (W & R)

Average number of steps: 8.6
Minimum number of steps: 8
Maximum number of steps: 17

Fig. 3. Sample derivation of Problem 408.10.2

the average number of [illegible] zero. Many students seemed to have terminated a session at this problem. The new rule used in the sample solution is HYP, to introduce the hypothesis of the desired conclusion.

Truth values. Exercises on assigning truth values to atomic formulas and on determining the truth value of a complex formula prepare the student for the task of finding counterexamples to arguments that can be symbolized in the propositional calculus. An example from Lesson 409, seen in Figure 4, shows the steps a student might take to decide on the validity of an argument.

Insert Figure 4 about here

Our tabulation of problems in which the student had a choice between constructing a counterexample or a derivation shows that this particular problem was attempted 37 times, 17 of which DER (derive) was correctly chosen first, 12 times as the second choice. The derivation is tricky because one premise is not needed in the solution. New rules used are form-a-disjunction (FD) and commute-a-disjunction (CD).

Identities. By Lesson 411, the student constructs proofs using rules for identity: number definition (ND), commute-equals (CE), and addition of equal terms (AE). Distribution of steps in the example, shown in Figure 5, is especially scattered, possibly because the solution can deceptively be constructed without a conditional proof. Only

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Insert Figure 5 about here

```
DERIVE        NOT (R)

P             (1) Q
P             (2) (S & Q) → NOT W
P             (3) W

CONSTRUCT A DERIVATION OR FIND A COUNTEREXAMPLE
TYPE  DER  OR  CEX
CEX
FIND A COUNTEREXAMPLE:
              NOT (R)
P             (1) Q
P             (2) (S & Q) → NOT W
P             (3) W

YOU ARE LOOKING FOR A COUNTEREXAMPLE.  THUS, THE
CONCLUSION MUST BE FALSE AND THE PREMISES TRUE.

NOW ASSIGN CONSISTENT TRUTH VALUES TO THE
REMAINING SENTENCE LETTERS.
S             F
W             T
Q             T
R             T

LET'S CHECK TO SEE IF YOUR TRUTH ASSIGNMENT
MAKES THE PREMISES TRUE AND THE CONCLUSION FALSE.
S & Q         F
NOT (W)       F
(S & Q) → NOT W     T

YOUR ASSIGNMENT MAKES ALL THE PREMISES TRUE.

LET'S CHECK THE CONCLUSION.
NOT (R)       F
CORRECT
```

Average number of steps: 8.1
Minimum number of steps: 7
Maximum number of steps: 12

Fig. 4. Sample derivation of Problem 409.16

DERIVE $(B = C \& C = D) \rightarrow D = B$

P	(1)	$(D = C \& C = B) \rightarrow D = B$
1CE2	(2)	$(D = C \& B = C) \rightarrow D = B$
2CE1	(3)	$(C = D \& B = C) \rightarrow D = B$
3CC1	(4)	$(B = C \& C = D) \rightarrow D = B$

Average number of steps: 7.8
Minimum number of steps: 4
Maximum number of steps: 17

Fig. 5. Sample derivation of Problem 411.14.2

one of the fall students, but five spring students, found the four-step solution.

Replacement of a term by an equal term (RE) is introduced in Lesson 413; more practice is given with this new rule in Lesson 414. Only one spring student and several fall students found the clever eight-step solution in the example, shown in Figure 6. The proof requires the rule of number definition.

Insert Figure 6 about here

Axioms. The commutative axiom (CA) is presented in Lesson 415 and the associative law for addition in Lesson 416. As our analysis of rules shows, AR and AL (associate right and left over addition) were difficult rules to use (as were the associative laws for Boolean algebra). Two problems from Lesson 416 are presented in Figures 7 and 8. We

Insert Figures 7 and 8 about here

selected them also because they are typical of the symbolic manipulations performed in constructing proofs in elementary algebra and in the propositional calculus. The minimum proof in the second example from this lesson does not require use of the AL rule, despite the fact that AL was introduced immediately before presentation of the problem. Several examples of such improper ordering of problems--improper in the sense that their context is misleading--have been corrected in later versions of the course.

Three more axioms are taught in Lesson 417: zero (Z), negative number (N), and additive inverse (AI). The example from this lesson is

DERIVE	NOT D < B	
P	(1)	D = 7
P	(2)	6 = B
P	(3)	5 + 1 = B → NOT 7 < B
ND6	(4)	6 = 5 + 1
2.4RE1	(5)	5 + 1 = B
3.5AA	(6)	NOT 7 < B
1CE1	(7)	7 = D
6.7RE1	(8)	NOT D < B

Average number of steps: 9.1
Minimum number of steps: 8
Maximum number of steps: 14

Fig. 6. Sample derivation of Problem 414.7.2

DERIVE 8 = (2 + 1) + 5

ND8	(1)	8 = 7 + 1
ND7	(2)	7 = 6 + 1
1.2RE1	(3)	8 = (6 + 1) + 1
ND6	(4)	6 = 5 + 1
3.4RE1	(5)	8 = ((5 + 1) + 1) + 1
5AR1	(6)	8 = (5 + (1 + 1)) + 1
ND2	(7)	2 = 1 + 1
7CE1	(8)	1 + 1 = 2
6.8RE1	(9)	8 = (5 + 2) + 1
9AR1	(10)	8 = 5 + (2 + 1)
8CA1	(11)	8 = (2 + 1) + 5

Average number of steps: 12.5
Minimum number of steps: 11
Maximum number of steps: 17

Fig. 7. Sample derivation of Problem 416.19.2

DERIVE $(5 + 1 = 4 + 2) \rightarrow 3 + (2 + 1) = 6$

HYP	(1)	$5 + 1 = 4 + 2$
ND6	(2)	$6 = 5 + 1$
2.1RE1	(3)	$6 = 4 + 2$
3CE1	(4)	$4 + 2 = 6$
ND4	(5)	$4 = 3 + 1$
4.5RE1	(6)	$(3 + 1) + 2 = 6$
6AR1	(7)	$3 + (1 + 2) = 6$
7CA2	(8)	$3 + (2 + 1) = 6$
1.8CP	(9)	$(5 + 1 = 4 + 2) \rightarrow 3 + (2 + 1) = 6$

Average number of steps: 13.6
Minimum number of steps: 9
Maximum number of steps: 25

Fig. 8. Sample derivation of Problem 416.22.2

interesting because the wide distribution of number of steps might have resulted from the inability to use the short forms of the axioms. The short form for N (apply N in one step by letting the program determine the correct instantiation of the axiom) is explained just before the example problem, shown in Figure 9, is assigned.

Insert Figure 9 about here

Theorems. Students first learn about theorems and their use in constructing proofs in Lesson 418. The example problem, shown in Figure 10, is an extremely simple one, yet only two students found the four-step

Insert Figure 10 about here

solution. They had just proved Theorem 1 $((-B) + B = 0)$, and had been encouraged to use it in subsequent proofs. The problem is misplaced, but we are still surprised at the difficulties most students seemed to have.

Lesson 420 is included in the curriculum to demonstrate a second axiomatic system, one in which the basic binary operation is noncommutative. We added this lesson in the winter, 1973, because in our previous experience we seemed to find students finishing with the impression that "the world commutes." Theorems proved in Lessons 418 and 419 must be proved in this new system; the proofs, without CA available, are of course more difficult than when CA is available.

Problem 420.4, shown in Figure 11, is the proof of the first theorem. It is, in terms of the number of hints requested by the students, the

Insert Figure 11 about here

DERIVE C + (3 + (-C)) = 1 + (1 + 1)

AI		B + -B = 0
B:C	(1)	C + -C = 0
ND3	(2)	3 = 2 + 1
ND2	(3)	2 = 1 + 1
2.3RE1	(4)	3 = (1 + 1) + 1
4CA2	(5)	3 = 1 + (1 + 1)
5AE		
:C + -C	(6)	3 + (C + -C) = (1 + (1 + 1)) + (C + -C)
6AI2	(7)	3 + (C + -C) = (1 + (1 + 1)) + 0
7Z1	(8)	3 + (C + -C) = 1 + (1 + 1)
8AL1	(9)	(3 + C) + -C = 1 + (1 + 1)
9CA1	(10)	(C + 3) + -C = 1 + (1 + 1)
10AR1	(11)	C + (3 + -C) = 1 + (1 + 1)

Average number of steps: 12.6
Minimum number of steps: 11
Maximum number of steps: 20

Fig. 9. Sample derivation of Problem 417.24.1

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DERIVE (0 + 6) + (11 - 5) = 6 + (11 - (5 + 0))

LT :(+ 6) + (11 - (5 + 0))	(1)	(0 + 6) + (11 - (5 + 0)) = (0 + 6) + (11 - (5 + 0))
1Z1	(2)	(0 + 6) + (11 - 5) = (0 + 6) + (11 - (5 + 0))
2CA3	(3)	(0 + 6) + (11 - 5) = (6 + 0) + (11 - (5 + 0))
3Z1	(4)	(0 + 6) + (11 - 5) = 6 + (11 - (5 + 0))

Average number of steps: 10.5
Minimum number of steps: 4
Maximum number of steps: 34

Fig. 10. Sample derivation of Problem 418.8.1

```
PROVE   (-B) + B = 0

AI   B + -B = 0
B:-B      (1)  (-B) + (--B) = 0
Z  B + 0 = B
B:-B      (2)  -B + 0 = -B
AI
B:B       (3)  B + -B = 0
3CE1      (4)  0 = B + -B
2.4RE1    (5)  (-B) + (B + -B) = -B
5AE
:--B      (6)  ((-B) + (B + -B)) + --B = (-B) + (--B)
6AL1      (7)  ((-B + B) + -B) + --B = (-B) + (--B)
7AR2      (8)  (-B + B) + ((-B) + --B) = (-B) + (--B)
8AI1      (9)  (-B + B) + 0 = (-B) + (--B)
9Z1      (10)  (-B) + B = (-B) + (--B)
10AI1    (11)  (-B) + B = 0
```

```
Average number of steps:   13.8
Minimum number of steps:    7
Maximum number of steps:   34
```

Fig. 11. Sample derivation of Problem 420.4.0

most difficult problem in the curriculum. An average of approximately two hints were requested, higher than any other problem. When this theorem was proved in the theory of Abelian groups, the average number of steps for a solution was two, in contrast to the average of 13 steps required in proving the same theorem without the commutative axiom. This same difficulty held true in proving the remaining six theorems in the lesson, despite the fact that availability of Theorem 1 should have made the proofs as simple as when CA was available.

Translation. We show only one example from Lesson 422 on translating English statements into the formalism of first-order logic (without quantifiers). It is a problem in which the student has already translated each premise and the conclusion as separate problems (Figure 12). The

Insert Figure 12 about here

sample proof uses De Morgan's Law (DM), definition of implications (DFA), deny-a-disjunct (DD), and right conjunct (RC).

Quantifiers. The notation we have developed for teletypewriter representation of "for all x, S(x)" and "there exists an x, S(x)" is "(A X)S(X)" and "(E X)S(X)", respectively. The commands for introducing and eliminating quantifiers (Lesson 426) are universal specification (US), universal generalization (UG), existential specification (ES), and existential generalization (EG). Restrictions on the use of these commands are taught in Lesson 427.

The rules US and UG are used in the next example problem, Figure 13, in which the student shows the validity of an argument about featherless

Insert Figure 13 about here

Q = LOVE IS BLIND
R = MEN ARE AWARE OF THE FACT THAT LOVE IS BLIND
S = WOMEN TAKE ADVANTAGE OF THE FACT THAT LOVE IS BLIND

DERIVE S

P	(1)	(Q & NOT R) OR (Q & S)
P	(2)	(NOT R) → NOT Q
2DFA	(3)	R OR NOT Q
3DM	(4)	NOT (NOT R & Q)
4CC1	(5)	NOT (Q & NOT R)
1.5DD	(6)	Q & S
6RC	(7)	S

Average number of steps: 10.5
Minimum number of steps: 7
Maximum number of steps: 20

Fig. 12. Sample derivation of Problem 422.39.0

ONLY BIRDS HAVE FEATHERS
NO MAMMAL IS A BIRD
.: EACH MAMMAL IS FEATHERLESS

DERIVE (A X) (M(X) → NCT F(X))

P	(1)	(A X) (F(X) → D(X))
P	(2)	(A X) (M(X) → NOT D(X))
1US X:X	(3)	F(X) → D(X)
2US X:X	(4)	M(X) → NOT D(X)
3CON	(5)	NOT D(X) → NOT F(X)
4.5HS	(6)	M(X) → NOT F(X)
7UG :X	(7)	(A X) (M(X) → NOT F(X))

Average number of steps: 9.1
Minimum number of steps: 7
Maximum number of steps: 17

Fig. 13. Sample derivation of Problem 426.44.0

mammals. It is the last problem in the Lesson. The minimum seven-step proof uses the contrapositive rule (CON), one not especially emphasized in the curriculum.

In the case of ES, we adopted the convention that the variable of quantification may only be replaced by an 'ambiguous' name. Ambiguous or arbitrary names are denoted by an asterisk followed by any variables occurring free in the formula to which ES is applied.

The rule EG replaces all occurrences of a proper name or ambiguous name by a variable of quantification. We chose problem 33 from Lesson 427 (Figure 14) to demonstrate the use of the four quantifier rules.

Insert Figure 14 about here

The problem is not necessarily a difficult one, making the wide distribution of the number of steps to solution somewhat surprising. The fall students did much better than the spring students on this problem.

In reviews requested during work on derivation problems, the program provides references to any flagged variables (i.e., those variables introduced free in a premise line) occurring in each line, as well as references to the premise line upon which the flagging depends. Again, this notation follows the rule usage adopted in Introduction to Logic (Suppes, 1957).

Interpretation. The notion of interpretation of a set of statements written in English is introduced in Lesson 423 and then elaborated on in Lesson 428 where the method for showing an argument invalid is explained. We take the set of rational numbers as the domain of interpretation. To show that an argument is invalid, the student provides an

DERIVE (E X) (I(X) & (A Y) (P(Y) → NOT L(X,Y)))

P	(1)	(A X) (P(X) → H(X))
P	(2)	(E X) (I(X) & (A Y) (H(Y) → NOT L(X,Y)))
2ES X:*	(3)	I(*) & (A Y) (H(Y) → NOT L(*,Y))
1US X:Y	(4)	P(Y) → H(Y)
3RC	(5)	(A Y) (H(Y) → NOT L(*,Y))
5US Y:Y	(6)	H(Y) → NOT L(*,Y)
4.6HS	(7)	P(Y) → NOT L(*,Y)
7UG :Y	(8)	(A Y) (P(Y) → NOT L(*,Y))
3LC	(9)	I(*)
9.8FC	(10)	I(*) & (A Y) (P(Y) → NOT L(*,Y))
10EG *:X	(11)	(E X) (I(X) & (A Y) (P(Y) → NOT L(X,Y)))

Average number of steps: 12.8
Minimum number of steps: 11
Maximum number of steps: 20

Fig. 14. Sample derivation of Problem 427.33.2

interpretation of each atomic formula in the argument and then proves the truth of an interpretation of a premise by deriving it as a theorem in the algebra of rational numbers. To prove a conclusion is false, the student derives the negation of the interpretation of the conclusion.

The student facilitates the repeated need to prove such statements as (A X)(NOT X=X → X=X) and (A X)(X=X → X=X) by using the system's INIT mode to prove lemmas. The ability to predict the usefulness of such lemmas shows a good level of understanding on the part of the better students in the course. We discuss this further in a later section.

In addition to constructing derivations to show that an argument is valid or invalid, we use derivations to show that an axiom is dependent on other axioms or that a premise is inconsistent with other premises. We use an example from Lesson 429 (Figure 15) to demonstrate the format of such problem types. The students show that a premise is

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Insert Figure 15 about here
----

inconsistent with other premises if its negation is derivable from the other premises. A set of premises is consistent if and only if it has at least one true interpretation (in terms of the axioms and theorems of elementary algebra). For example, students construct a proof that the two premises

All unicorns are animals.

No unicorns are animals.

are consistent; in the figure, the student proves Axiom 1 is inconsistent

ALL MEN ARE ANIMALS
ALL ANIMALS ARE MORTAL
SOME MEN ARE NOT MORTAL

SHOW THE THIRD PREMISE IS INCONSISTENT
WITH THE FIRST TWO PREMISES

DERIVE NOT (E X) (H(X) & NOT M(X))

P	(1)	(A X) (H(X) → S(X))
P	(2)	(A X) (S(X) → M(X))
1UG X:X	(3)	H(X) → S(X)
2UG X:X	(4)	S(X) → M(X)
3.4HS	(5)	H(X) → M(X)
5DFA	(6)	NOT H(X) OR M(X)
6DM	(7)	NOT (H(X) & NOT M(X))
7UG :X	(8)	(A X) (NOT (H(X) & NOT M(X)))
8QN	(9)	NOT (E X) (H(X) & NOT M(X))

Average number of steps: 11.3
Minimum number of steps: 8
Maximum number of steps: 18

Fig. 15. Sample derivation of Problem 429.11.0

with Axioms 2 and 3 by constructing a proof in which the first axiom is false and the other two true.

We selected one problem from the lessons on Boolean algebra (Figure 16). The average number of hints for this problem was one; two hints

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Insert Figure 16 about here
----

were actually available. This is proof of Theorem 179. Data from the next three theorems also showed a high number of requests for hints.

The symbolization problems in Lesson 435 present another challenge to the students' intuitive reasoning. If the student's response is not logically equivalent to a stored answer, he must construct a proof to show that logical equivalence does not hold between his answer and the one stored with the curriculum problem. He must also designate which of the two expressions will have the false interpretation and which the true, since the implication may hold only in one direction. To carry out proofs in this lesson, students were encouraged to prove (NOT NOT R) IFF R, which should have made problem 435.13 (Figure 17), an exercise in

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Insert Figure 17 about here
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proving two statements not logically equivalent, quite easy. Judging from the scattered distribution of solution steps, this suggestion did not seem to help. This problem was also the first time RQ, replace-equivalent-formulas, was available for use by the students. We show two solutions--one using quantifier rules, the other using RQ.

Finding axioms. In a previous report (Goldberg & Suppes, 1972) we examined the diversity of solutions obtained by students for the

THEOREM 179

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PROVE  (G ∨ H = G ↑ H) → G = H

HYP          (1)      G ∨ H = G ↑ H
TH168   G ↑ (G ∨ H) = G
G:G
H:H          (2)      G ↑ (G ∨ H) = G
2.1RE1       (3)      G ↑ (G ↑ H) = G
IA   (G ↑ H) ↑ K = G ↑ (H ↑ K)
G:G
H:G
K:H          (4)      (G ↑ G) ↑ H = G ↑ (G ↑ H)
4.3RE1       (5)      (G ↑ G) ↑ H = G
5TH164.1     (6)      G ↑ H = G
TH168   (G ↑ (G ∨ H)) = G
G:H
H:G          (7)      (H ↑ (H ∨ G)) = H
1CU1         (8)      H ∨ G = G ↑ H
7.8RE1       (9)      H ↑ (G ↑ H) = H
9CI1        (10)      (G ↑ H) ↑ H = H
10IA1       (11)      G ↑ (H ↑ H) = H
11TH164.1   (12)      G ↑ H = H
12.6RE1     (13)      G = H
1.13CP      (14)  (G ∨ H = G ↑ H) → G = H
```

Average number of steps: 21.5
Minimum number of steps: 14
Maximum number of steps: 38

Fig. 16. Sample derivation of Problem 423.44.0

...

DERIVE NOT (A X) D(X)

Proof 1:	P	(1)	NOT (A X) NOT NOT D(X)	
	WP	(2)	D(X)	(STEPS 2-8 ARE A LEMMA.)
	2DN	(3)	NOT NOT D(X)	
	2.3CP	(4)	D(X) → NOT NOT D(X)	
	WP	(5)	NOT NOT D(X)	
	5DN	(6)	D(X)	
	5.6CP	(7)	NOT NOT D(X) → D(X)	
	7.4LB	(8)	NOT NOT D(X) IFF D(X)	
	1.8RQ1	(9)	NOT (A X) D(X)	

Proof 2:	P	(1)	NOT (A X) NOT NOT D(X)
	HYP	(2)	(A X) D(X)
	1QNC	(3)	(E X) NOT D(X)
	3ES X:*	(4)	NOT D(*)
	2US X:*	(5)	D(*)
	4.5.2IP	(6)	NOT (A X) D(X)

Average number of steps: 9.8
Minimum number of steps: 3
Maximum number of steps: 19

Fig. 17. Sample derivation of Problem 434.13.0

finding-axioms exercises. The students were seventh graders as well as college students. The data for the classes analyzed here were broadly similar, and consequently we omit all details.

Choice of method. Three kinds of problems in the curriculum are designed to give the student an opportunity to develop some intuition about the validity or invalidity of arguments. They are (1) derive or find a counterexample using truth table analysis (DC), (2) derive or find a counterexample using the method of interpretation (DI), and (3) show premises inconsistent or give an interpretation to show premises inconsistent (DIC). In each case, the student states his choice by typing commands DER (derive), CEX (counterexample by truth analysis), or INT (method of interpretation). He can change his choice at any time by typing one of the three choices; he can restart within the present choice by typing RESTART.

Table IX presents our data on the students' choices, sorted by problem number. It provides an enumeration of the number of times each

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Insert Table IX about here
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problem was attempted, the correct choice, and the number of times the choice was correct on the first or the second try. Table X compiles

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Insert Table X about here
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the same data, sorting by problem type. There are 27 problems (15 with random selection) involved. From this information, it appears that the students have some difficulty with each type of problem, some of which we feel was due to defects in our presentation of uses of the method of interpretation.

TABLE IXa

Data on Students' Choices of Correct Method of Attack (Spring, 1973)

Problem No.	Problem type	Correct choice	No. times chosen first	No. times chosen second	Total times problem tried
409.12.1	DC	DER	15	8	24
409.12.2	DC	CEX	10	8	24
409.13.1	DC	DER	16	6	22
409.13.2	DC	CEX	13	4	17
409.14.1	DC	CEX	16	4	27
409.14.2	DC	CEX	19	5	31
409.15.1	DC	CEX	20	3	24
409.15.2	DC	CEX	16	0	20
409.16.1	DC	CEX	19	6	26
409.16.2	DC	DER	9	4	17
409.17.1	DC	CEX	23	2	26
409.17.2	DC	CEX	11	4	18
409.18.0	DC	DER	3	0	3
409.19.1	DC	CEX	14	3	20
409.19.2	DC	CEX	5	2	24
409.20.1	DC	DER	9	5	16
409.20.2	DC	CEX	21	7	29
409.21.1	DC	CEX	21	1	22
409.21.2	DC	CEX	18	0	20
409.26.1	DC	DER	31	5	39
409.26.2	DC	DER	13	14	35
428.17.0	DI	INT	29	10	45
428.18.1	DI	INT	12	1	23
428.18.2	DI	INT	13	4	21
428.19.1	DI	INT	9	2	13
428.19.2	DI	DER	12	1	26
428.20.0	DI	INT	1[illegible]	3	16
428.21.1	DI	DER	20	1	23
428.21.2	DI	DER	7	0	9

(Table IXa, cont.)

Problem No.	Problem type	Correct choice	No. times chosen first	No. times chosen second	Total times problem tried
428.22.1	DI	DER	13	0	15
428.22.2	DI	DER	17	0	22
428.23.1	DI	INT	9	9	19
428.23.2	DI	INT	11	3	15
428.24.0	DI	INT	11	0	12
428.25.0	DI	INT	14	5	20
428.26.0	DI	DER	14	0	17
429.20.0	DI	INT	30	16	50
429.21.0	DI	INT	52	22	86
429.22.0	DI	INT	33	10	55
429.12.0	DIC	INT	28	12	46
429.13.0	DIC	DER	28	5	41
429.14.0	DIC	INT	18	20	39

TABLE IXb

Data on Students' Choices of Correct Method of Attack (Fall, 1973)

Problem No.	Problem type	Correct choice	No. times chosen first	No. times chosen second	Total times problem tried
409.12.1	DC	DER	13	8	21
409.12.2	DC	CEX	8	8	18
409.13.1	DC	DER	10	8	22
409.13.2	DC	CEX	11	6	18
409.14.1	DC	CEX	14	7	22
409.14.2	DC	CEX	15	2	18
409.15.1	DC	CEX	15	6	21
409.15.2	DC	CEX	13	2	16
409.16.1	DC	CEX	17	3	20
409.16.2	DC	DER	8	8	20
409.17.1	DC	CEX	15	4	19
409.17.2	DC	CEX	14	2	16
409.18.0	DC	DER	1	0	3
409.19.1	DC	CEX	18	1	22
409.19.2	DC	CEX	4	1	18
409.20.1	DC	DER	17	3	22
409.20.2	DC	CEX	17	4	24
409.21.1	DC	CEX	24	0	25
409.21.2	DC	CEX	13	1	14
409.26.1	DC	DER	26	12	44
409.26.2	DC	DER	17	13	36
428.17.0	DI	INT	27	16	47
428.18.1	DI	INT	16	2	20
428.18.2	DI	INT	11	9	21
428.19.1	DI	INT	16	6	22
428.19.2	DI	DER	7	0	28
428.20.0	DI	INT	17	0	20
428.21.1	DI	DER	22	0	27
428.21.2	DI	DER	14	2	19

(Table IXb, cont.)

Problem No.	Problem type	Correct choice	No. times chosen first	No. times chosen second	Total times problem tried
428.22.1	DI	DER	20	1	22
428.22.2	DI	DER	15	0	18
428.23.1	DI	INT	11	7	23
428.23.2	DI	INT	19	3	23
428.24.0	DI	INT	8	2	17
428.25.0	DI	INT	9	5	15
428.26.0	DI	DER	12	3	18
429.20.0	DI	INT	33	14	58
429.21.0	DI	INT	43	15	64
429.22.0	D[illegible]	INT	37	9	49
429.12.0	[illegible]	INT	28	16	52
429.13.0	[illegible]C	DER	41	5	55
429.14.0	DIC	INT	15	23	47

TABLE Xa

Choice of Method of Attack by Problem Type (Spring, 1973)

Problem type	No. of first choices correct	% first choice correct	No. of second choices correct	% second* choice correct	No. of times type occurs
DC	322	66.5	91	56.1	484
DI	316	64.8	87	50.9	487
DIC	74	58.7	37	71.2	126

*Given first choice incorrect.

TABLE Xb

Choice of Method of Attack by Problem Type (Fall, 1973)

Problem type	No. of first choices correct	% first choice correct	No. of second choices correct	% second* choice correct	No. of times type occurs
~~DC~~	~~290~~	~~66.1~~	~~99~~	~~66.4~~	~~439~~
DI	337	65.9	94	54.0	511
DIC	84	54.5	44	62.9	154

*Given first choice incorrect.

Special Commands

Several special features of the instructional system are of interest: hints, reviews, initiative, and redoing solutions. In the previous section we displayed three of the ten problems in which the average number of hints requested was approximately one, or, in one case only, two. Students reported that they did not always ask for a hint when they needed one (seeking a teaching assistant instead) because they were not aware they were able to do so or, when they did, no hints were available. We found that the students in the spring quarter had a 57 percent error rate with the HINT command; 1,291 out of 2,232 uses of HINT were handled with the comment "NO HINT AVAILABLE." In the fall, the error rate for INIT was 54 percent, that is, 1,324 out of 2,449 uses of the command. Detailed data on all problems which either had a stored hint or for which at least one student requested a hint are shown in Table XI. An extended

Insert Table XI about here

discussion of the problem of helping students with hints and other methods is given in Goldberg (1973).

A simple visual aid, reprinting the partial solutions with error messages and deleted lines omitted, was provided when the student typed the command REVIEW. REVIEW was used on the average about 44 times in the spring quarter and about 69 times in the fall quarter. Several students were recorded as requesting a review of their work more than 100 times. In other cases, students never used the command. This

TABLE XIa

Data on Availability and Use of Hints (Spring, 1973)

Problem no.	No. of hints available	No. of hints requested	Problem no.	No. of hints available	No. of hints requested
404.18.1	0	1	408.12.1	1	6
404.20.1	0	3	408.12.2	1	13
404.22.1	2	10	408.13.1	1	6
404.22.2	2	12	408.13.2	1	0
405.2.1	0	1	408.14.1	1	11
405.2.2	0	1	408.14.2	0	5
405.3.1	0	2	408.15.1	1	12
405.3.2	0	4	408.15.2	1	16
405.4.1	0	13	408.16.2	0	4
405.4.2	0	4	408.17.1	0	1
405.5.1	0	8	408.17.2	0	2
405.7.1	0	11	408.18.1	0	3
405.7.2	0	2	408.18.2	0	1
405.8.1	0	6	408.19.1	1	10
405.8.2	0	7	408.20.0	1	7
405.10.1	0	7	408.21.0	0	4
405.10.2	0	1	408.22.0	2	17
405.11.1	0	3	408.23.1	1	4
405.11.2	0	5	408.23.2	1	15
405.12.1	0	6	408.24.1	0	5
405.12.2	0	4	408.24.2	0	5
406.4.2	0	1	408.25.1	0	8
406.18.2	0	1	408.25.2	0	2
407.1.0	0	1	408.26.1	0	15
407.4.1	0	1	408.26.2	0	11
407.14.1	0	1	408.27.0	0	4
407.15.2	0	1	408.28.0	0	2
407.20.1	0	2	409.1.0	0	1
407.21.1	0	5	409.12.1	0	2
407.21.2	0	5	409.13.1	0	1
408.6.0	0	14	409.16.2	0	1
408.7.1	0	2	409.20.1	0	3
408.8.1	0	18	409.24.1	1	4
408.8.2	0	15	409.24.2	1	11
408.9.1	0	5	409.25.1	0	33
408.9.2	0	6	409.25.2	0	15
408.10.1	3	68	409.26.1	0	4
408.10.2	3	45	409.26.2	0	9
408.11.1	1	10	410.16.1	0	2
408.11.2	1	10	410.16.2	0	4

(Table XIa, cont.)

Problem no.	No. of hints available	No. of hints requested	Problem no.	No. of hints available	No. of hints requested
410.17.2	0	1	418.18.1	0	1
410.26.2	0	1	418.19.1	0	1
410.28.1	0	3	418.21.2	0	1
410.29.2	0	1	418.25.1	0	1
411.14.2	0	2	418.25.2	0	3
412.24.1	0	1	419.3.2	0	1
412.24.2	0	1	419.11.2	0	1
412.27.2	0	1	419.13.1	0	1
414.2.1	0	1	420.4.0	0	95
414.2.2	0	1	420.5.0	0	21
414.5.1	0	1	420.6.0	0	1
414.8.2	0	1	420.8.0	0	2
414.12.1	0	1	420.9.0	0	11
414.14.2	0	1	420.10.0	1	32
415.21.2	0	6	422.19.0	0	2
415.24.2	0	1	422.23.0	0	1
415.25.1	0	1	422.27.0	0	1
415.25.2	0	6	422.28.0	0	2
415.26.1	0	5	422.39.0	0	2
415.26.2	0	2	424.11.0	0	3
415.31.2	0	2	424.12.0	0	1
415.32.2	0	1	426.4.0	0	1
416.8.1	0	1	426.6.0	0	2
416.8.2	0	1	426.9.1	0	1
416.9.1	0	3	426.11.1	0	1
416.9.2	0	3	426.11.2	0	2
416.14.0	1	0	426.32.2	0	2
416.18.2	0	2	426.38.2	0	5
416.19.1	0	1	427.31.0	0	4
416.19.2	0	1	427.32.1	0	2
417.17.1	0	1	427.32.2	0	5
417.18.2	0	1	427.33.1	0	2
417.19.2	0	1	427.33.2	0	1
417.25.1	0	2	427.35.0	2	0
417.25.2	0	1	427.36.0	3	17
417.26.2	0	1	427.37.0	1	0
417.27.1	1	10	427.38.0	1	2
418.3.0	0	2	427.40.0	1	5
418.8.1	0	2	427.42.0	0	1
418.8.2	0	2	427.49.0	0	8
418.10.2	0	1	428.13.0	0	29
418.14.1	0	2	428.14.0	0	3
418.17.1	0	4	428.15.0	0	2

(Table XIa, cont.)

Problem no.	No. of hints available	No. of hints requested	Problem no.	No. of hints available	No. of hints requested
428.17.0	0	1	432.25.0	1	15
428.20.0	0	2	432.27.0	1	33
428.23.1	0	2	432.28.0	0	16
~~429.6.0~~	~~0~~	~~1~~	~~432.30.0~~	~~3~~	~~43~~
429.12.0	0	4	432.33.0	0	8
429.13.0	0	2	432.34.0	0	3
429.14.0	0	2	432.37.0	2	45
429.17.0	0	5	432.38.0	3	45
429.18.0	1	6	432.41.0	0	10
429.20.0	1	21	432.42.0	0	2
429.21.0	2	72	432.43.0	0	2
429.22.0	0	36	432.44.0	2	42
431.2.0	0	1	432.45.0	2	39
431.7.0	0	3	432.46.0	2	74
431.9.0	0	3	432.47.0	2	55
431.17.0	0	3	433.5.0	0	1
431.20.0	1	1	433.6.0	0	2
431.21.0	0	2	433.7.0	0	14
431.25.0	0	2	433.8.0	2	51
431.26.0	0	6	433.9.0	1	25
431.33.0	0	3	433.10.0	1	30
431.34.0	0	11	433.12.0	1	33
432.10.0	0	6	433.13.0	1	12
432.12.0	0	3	433.14.0	1	27
432.13.0	0	1	435.11.0	0	2
432.14.0	0	3	435.12.0	0	2
432.16.0	2	15	435.13.0	0	25
432.21.0	1	31	435.16.0	0	12
432.24.0	1	26	435.17.0	0	9

TABLE XII

Data on Availability and Use of Hints (Fall, 1973)

Problem no.	No. of hints available	No. of hints requested	Problem no.	No. of hints available	No. of hints requested
404.13.2	0	2	408.19.1	1	11
404.22.1	2	3	408.19.2	0	2
404.22.2	2	13	408.20.0	1	0
405.2.1	0	3	408.22.0	2	12
405.2.2	0	1	408.23.1	1	5
405.4.1	0	6	408.23.2	1	17
405.4.2	0	6	408.24.1	0	1
405.5.2	0	1	408.24.2	0	7
405.8.1	0	9	408.25.1	0	3
405.8.2	0	1	408.25.2	0	3
405.9.1	0	5	408.26.1	0	8
405.9.2	0	3	408.26.2	0	12
405.11.2	0	1	408.27.0	0	3
405.12.1	0	2	408.28.0	0	2
405.12.2	0	3	409.1.0	0	2
406.19.1	0	1	409.12.1	0	2
407.3.1	0	1	409.18.0	0	1
407.21.2	0	5	409.20.1	0	5
408.6.0	0	14	409.22.0	0	1
408.8.1	0	9	409.24.1	1	8
408.8.2	0	10	409.24.2	1	13
408.9.1	0	2	409.25.1	0	21
408.9.2	0	3	409.25.2	0	13
408.10.1	3	34	409.26.1	0	9
408.10.2	3	52	409.26.2	0	10
408.11.1	1	11	410.16.1	0	3
408.11.2	1	1	410.16.2	0	1
408.12.1	1	7	410.22.2	0	1
408.12.2	1	3	410.27.2	0	5
408.13.1	1	7	411.11.2	0	1
408.13.2	1	6	411.24.2	0	1
408.14.1	1	9	412.22.2	0	1
408.14.2	0	7	412.24.2	0	2
408.15.1	1	16	414.2.2	0	2
408.15.2	1	16	414.18.1	0	1
408.16.1	0	4	415.21.2	0	1
408.16.2	0	1	415.25.2	0	4
408.17.1	0	2	415.26.1	0	8
408.17.2	0	2	415.26.2	0	3
408.18.1	0	1	415.27.1	0	1
408.18.2	0	2	415.32.1	0	1

(Table XIb, cont.)

Problem no.	No. of hints available	No. of hints requested	Problem no.	No. of hints available	No. of hints requested
416.8.2	0	1	427.49.0	0	5
416.9.1	0	1	428.13.0	0	18
416.9.2	0	2	428.15.0	0	3
~~416.14.0~~	~~1~~	~~0~~	428.17.0	0	2
416.19.1	0	1	428.19.1	0	1
417.17.1	0	1	428.21.2	0	1
417.17.2	0	1	428.22.1	0	2
417.25.1	0	2	428.22.2	0	1
417.25.2	0	1	428.23.1	0	1
417.26.2	0	1	428.24.0	0	3
417.27.1	1	15	428.25.0	0	1
418.8.2	0	2	428.26.0	0	1
418.10.2	0	1	429.6.0	0	6
418.14.2	0	1	429.11.0	0	1
418.17.1	0	2	429.12.0	0	3
418.25.1	0	2	429.13.0	0	3
418.25.2	0	1	429.14.0	0	4
418.27.1	0	1	429.17.0	0	3
420.4.0	0	130	429.18.0	1	6
420.5.0	0	18	429.20.0	1	33
420.10.0	1	32	429.21.0	2	54
422.19.0	0	2	429.22.0	0	33
422.27.0	0	1	431.7.0	0	2
422.28.0	0	4	431.9.0	0	1
422.39.0	0	4	431.17.0	0	2
424.11.0	0	5	431.20.0	1	0
424.12.0	0	1	431.21.0	0	1
426.3.0	0	1	431.24.0	0	1
426.6.0	0	4	431.25.0	0	3
426.9.1	0	2	431.26.0	0	5
426.11.1	0	3	431.34.0	0	12
426.32.2	0	1	432.8.0	0	1
426.38.2	0	8	432.10.0	0	4
426.44.0	0	1	432.12.0	0	5
427.31.0	0	6	432.16.0	2	17
427.32.1	0	5	432.21.0	1	26
427.32.2	0	1	432.24.0	1	24
427.33.1	0	3	432.25.0	1	15
427.33.2	0	1	432.27.0	1	40
427.35.0	2	0	432.28.0	0	18
427.36.0	3	9	432.30.0	3	69
427.37.0	1	0	432.33.0	0	9
427.38.0	1	0	432.34.0	0	7
427.40.0	1	2	432.37.0	2	36

(Table XIb, cont.)

Problem no.	No. of hints available	No. of hints requested	Problem no.	No. of hints available	No. of hints available
432.38.0	3	52	433.8.0	2	55
432.41.0	0	12	433.9.0	1	32
432.42.0	0	1	433.10.0	1	38
~~432.43.0~~	~~0~~	~~2~~	~~433.12.0~~	~~1~~	~~32~~
432.44.0	2	66	433.13.0	1	14
432.45.0	2	48	433.14.0	1	31
432.46.0	2	43	435.11.0	0	2
432.47.0	2	83	435.13.0	0	17
433.5.0	0	1	435.16.0	0	9
433.6.0	0	1	435.17.0	0	6
433.7.0	0	4			

information is tabulated in Table XII. Another way to get a fresh start

----
Insert Table XII about here
----

on a solution is to delete all lines except premises (which are, of course, part of the problem statement). Out of 7,577 (8,251 in the fall) uses of the rule DLL (delete the last lines), 2,017 (2,601) were requests to restart a proof--about 27 percent (32 percent in the fall). So this was a generally useful command.

The third feature is under curriculum control. It is possible to require certain commands to be used, or not used, in the construction of a proof. If the student completes a solution that is incorrect solely because it does not meet the constraints, he is asked to redo the problem. This is utilized in the curriculum to

(1) get the student to use a newly taught rule or recently proved theorem;

(2) encourage the student to try diverse solutions; and

(3) encourage use of short forms of rules by constraining the use of replace-equals rule (RE).

Students in both quarters had to redo a solution only about 3 times out of 35 possible problems that included such constraints; only 4 students avoided the need to redo a problem.

Many students had difficulty finding a solution within the constraints of problem 415.30 (ostensibly chosen to encourage the use of the short form of CA):

```
USE LT IN THIS PROBLEM.
DERIVE  5+6=6+5
```

TABLE XIIa

Student Use of Special Commands

(Data by Individual Students, Spring, 1973)

Student	NEWS	INIT	LESSON	REVIEW	REDO
2850	13	75	49	28	1
2851	2	6	0	0	4
2852	9	10	0	145	2
2853	2	31	18	89	4
2854	1	8	0	35	2
2855	8	25	29	94	2
2856	5	60	7	165	7
2857	8	20	5	32	3
2858	5	22	37	79	2
2859	7	27	20	14	2
2860	2	8	3	12	0
2861	6	24	22	7	2
2863	9	6	0	84	3
2864	3	14	24	67	4
2865	4	10	7	0	3
2867	5	26	15	61	1
2868	4	7	1	65	3
2869	3	7	0	89	7
2870	7	30	19	0	0
2871	4	13	15	9	2
2872	20	45	17	80	3
2873	3	20	1	46	3
2875	6	24	18	15	2
2877	2	11	3	73	4
2878	5	11	0	0	5
2879	9	31	10	26	1
2880	1	18	14	59	4
2881	7	14	2	5	5
2882	1	30	13	43	2
2883	12	33	9	0	3
2884	4	5	0	0	2
2886	6	6	3	0	4
2888	1	15	0	21	2
2889	20	35	37	61	3
2890	0	21	19	23	1
2891	6	22	12	0	2
2892	4	1	0	0	3
2893	12	30	0	154	8
2894	3	17	1	29	2
2896	5	17	11	33	4
2897	10	16	0	0	3
2898	5	14	21	79	2
Totals	249	865	462	1822	122
Average	5.9	20.6	11.0	43.4	2.9

TABLE XIIb

Student Use of Special Commands

(Data by Individual Students, Fall, 1973)

Student	NEWS	INIT	LESSON	REVIEW	REDO
1301	22	32	14	217	3
1302	8	6	0	56	4
1303	11	13	17	27	3
1304	6	16	7	196	2
1305	10	33	29	39	7
1310	7	25	19	1	3
1311	6	22	6	27	0
1312	4	56	21	120	6
1313	4	20	6	137	4
1314	7	3	0	52	2
1316	16	61	22	189	2
1320	5	9	14	67	3
1322	4	27	25	68	4
1323	7	41	18	46	4
1324	5	22	16	47	4
1325	7	28	1	28	2
1328	20	36	15	90	4
1330	4	13	7	31	2
1331	7	17	1	2	1
1332	7	6	6	34	4
1333	7	24	5	3	2
1334	9	14	6	41	2
1335	9	16	19	63	0
1338	25	24	2	89	1
1342	6	31	15	31	1
1345	10	33	15	228	0
1346	2	14	16	35	4
1347	17	13	12	64	4
1348	4	9	2	38	6
1349	14	31	49	3	4
1352	1	22	14	53	4
1355	9	14	5	20	1
1356	9	22	14	107	4
1358	8	8	2	30	4
1359	12	18	9	271	3
1360	15	29	11	0	2
Totals	328	840	461	2694	115
Average	9.1	23.3	12.8	74.8	3.2

An example chosen to 'encourage' diverse solution paths is 418.25:

```
CONSTRUCT A DERIVATION WHICH DOES NOT USE CA.
YOU WILL HAVE TO USE AS AND ND.
DERIVE  3+2=2+3.
```

It is, frankly, not a very cleverly chosen problem; it is part of the planned revision to replace it.

Students received information about seminars and system schedules by typing NEWS. This command was infrequently used, an average of approximately 6 times (8 in the fall) in contrast to approximately 62 (37) average sessions at the terminal. Teaching assistants preferred to list and post the course news because printing news at the 10-character-per-second output rate was a time-consuming task for each student to undertake.

We have already mentioned the relatively large percentage of times LESSON was used to alter the sequence of problem presentations. Our scan of the protocol data indicates the command was used to skip ahead the majority of the times it was called upon. LESSON represents one of the commands students could type after obtaining the initiative (INIT command) to request their own problems. Another command was FA (to select a finding-axioms exercise). Two other commands, DERIVE and PROVE, are analyzed in a subsequent section.

Use of the Command Language

In order to carry out solutions to the curriculum problems, the students must learn to use a new language: a set of commands having a strict syntax and a semantics corresponding to that of the first-order predicate calculus with identity. Example problems demonstrated the use of this language. Each command has, at most, four parts:

(1) a list of references to previous lines of the derivation,

(2) a command name,

(3) a list of references to occurrences of terms or formulas, and

(4) requests for terms and formulas.

So, for example, we have (student input underlined):

1.2.3IP	indirect proof procedure, requiring 3 line references;
1AR2	associate right over addition, requiring a line reference and a reference to the occurrence of an instance of (B+C)+D;
1FD :R	form a disjunction, requiring a line reference and a request for a well-formed formula;
1ES X:*	existential specification, requiring a request for an ambiguous name;
1EG *:X	existential generalization, requiring both the ambiguous name and the variable of generalization;
NG (A X)(E Y)(X<Y) X: 5	an axiom, requiring instantiation of all universally quantified variables whose scope of quantification is the entire formula;
1TH5.2	short-form notation for a theorem (all theorem names are of the form TH followed by a number), requiring a line reference, theorem name, and reference to the occurrence

	in the line on the left-hand side of the theorem; and
<u>WP</u> (i) <u>R OR Q</u>	working premise, the only command that expects the student to type the actual line of the proof.

Several kinds of error messages could be sent to a student using a command improperly. Syntax-error messages described the correct number of line or occurrence references required, or stated if a formula or term was not well formed. Application-error messages commented on (a) attempts to use a command not learned yet or not proved; (b) inability to locate the referent of an occurrence number; (c) attempts to refer to a line that is no longer available because it is part of a completed subsidiary derivation; (d) attempts to use a line as a working premise which was not introduced as such; and (e) improper applications of the quantifier rules (US, UG, ES, EG) according to the restrictions taught in Lessons 426 and 427.

Table XIII itemizes each name entered in the command language by the curriculum author, the total number of times students used the command, and the number and percentage of times errors were noted in the

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Insert Table XIII about here

---

attempted use of the command. These errors are further broken down into the kinds of errors: syntax or application.

If we view the curriculum as providing lessons on and practice with manipulations of formulas in symbolic logic, the table presents no surprises. (Further support for taking this point of view is offered in

TABLE XIIIa **BEST COPY AVAILABLE**

Data on Rule Usage Summed over Students and Exercises (Spring, 1973)

Rule	No. of times rule used	No. of times error in use	Syntax errors	Appl. errors	% total errors
DFA	998	118	6	112	11
DNA	349	46	2	44	13
CON	196	10	5	5	5
AA	9201	911	151	760	9
CC	180	57	43	14	31
CD	575	180	141	39	31
CE	5853	348	306	42	5
CQ	8	4	2	2	50
AE	1905	80	64	16	4
SE	925	21	15	6	2
DIFF	0	0	0	0	0
DC	1030	165	21	144	16
DD	2145	270	37	233	12
DM	1443	115	20	95	7
DN	1436	40	40	0	2
DS	6	2	1	1	33
FC	2684	100	99	1	3
FD	1580	85	85	0	5
HS	304	23	3	20	7
LB	103	27	5	22	26
LC	4151	174	32	142	4
LT	4516	219	203	16	4
RC	3925	147	30	117	3
QNA	169	36	2	34	21
QNB	409	17	4	13	4
QNC	216	21	2	19	9
QND	424	22	5	17	5
CA	3165	129	72	57	4
AS	650	51	26	25	7
Z	1635	151	30	121	9
N	1490	140	21	119	9
AI	1195	119	22	97	9
U	113	5	5	0	4
NS	564	23	9	14	4
AD	8	1	0	1	12
TR	164	12	4	8	7
CN	134	5	4	1	3
DG	116	24	10	14	20
NL	567	5	3	2	0
NG	472	9	5	4	1
AR	1519	265	18	247	17
AL	748	154	16	138	20

(Table XIIIa, cont.)

Rule	No. of times rule used	No. of times error in use	Syntax errors	Appl. errors	% total errors
UI	774	62	13	49	8
CU	985	38	12	26	3
DI	821	136	34	102	16
DU	699	85	19	66	12
II	643	47	16	31	7
RA	740	44	9	35	5
EM	683	50	14	36	7
UA	27	10	1	9	37
UR	152	24	3	21	15
UL	88	14	1	13	15
SA	178	21	12	9	11
CS	161	8	3	5	4
IA	241	28	8	20	11
TH1	580	48	23	25	8
TH2	603	42	22	20	6
TH3	293	22	5	17	7
TH4	195	15	4	11	7
TH5	254	3	2	1	1
TH6	98	3	1	2	3
TH7	60	17	3	14	28
TH8	2	0	0	0	0
TH9	1	0	0	0	0
TH10	0	0	0	0	0
TH11	0	0	0	0	0
TH12	0	0	0	0	0
TH13	1	0	0	0	0
TH14	1	0	0	0	0
TH15	0	0	0	0	0
TH16	3	0	0	0	0
TH17	1	1	0	1	100
TH18	0	0	0	0	0
TH19	0	0	0	0	0
TH20	0	0	0	0	0
TH21	2	1	1	0	50
TH22	1	0	0	0	0
TH60	10	0	0	0	0
TH61	161	29	28	1	18
TH62	148	5	2	3	3
TH63	2	0	0	0	0
TH64	1	0	0	0	0
TH65	1	0	0	0	0
TH66	5	1	1	0	20
TH67	4	1	1	0	25
TH68	0	0	0	0	0
TH69	2	0	0	0	0

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(Table XIIIa, cont.)

Rule	No. of times rule used	No. of times error in use	Syntax errors	Appl. errors	% total errors
TH70	77	2	1	1	2
TH71	55	1	1	0	1
TH72	61	1	0	1	1
TH161	96	4	2	2	4
TH162	79	3	0	3	3
TH163	199	11	3	8	5
TH164	266	19	6	13	7
TH165	228	25	6	19	10
TH166	273	13	9	4	4
TH167	153	7	0	7	4
TH168	199	12	2	10	6
TH169	16	2	2	0	12
TH170	18	6	6	0	33
TH171	13	0	0	0	0
TH172	7	1	1	0	14
TH173	78	10	9	1	12
TH174	122	17	12	5	13
TH175	7	1	1	0	14
TH176	7	1	1	0	14
TH177	36	3	2	1	8
TH178	37	3	2	1	8
TH179	21	2	2	0	9
TH180	45	10	8	2	22
TH181	33	5	3	2	15
TH182	13	2	1	1	15
TH183	5	0	0	0	0
TH184	1	0	0	0	0
TH185	2	0	0	0	0
TH186	1	0	0	0	0
TH187	72	7	5	2	9
TH188	45	7	6	1	15
TH189	7	1	1	0	14
TH190	12	1	1	0	8
TH191	10	2	2	0	20
TH192	3	0	0	0	0
TH193	0	0	0	0	0
CP	13035	1209	317	892	9
DLL	7577	270	243	27	3
EG	2092	200	8	192	9
ES	3406	857	69	788	25
HINT	2232	1291	0	1291	57
HYP	7947	0	0	0	0
IP	4245	742	176	566	17
ND	2948	70	60	10	2

(Table XIIIa, cont.)

Rule	No. of times rule used	No. of times error in use	Syntax errors	Appl. errors	% total errors
RE	8824	816	190	626	9
REVIEW	1778	0	0	0	0
RQ	68	14	10	4	20
UG	3146	195	47	148	6
US	4421	302	47	255	6
WP	14702	15	15	0	0

TABLE XIIIb

Data on Rule Usage Summed over Students and Exercises (Fall, 1973)

Rule	No. of times rule used	No. of times error in use	Syntax errors	Appl. errors	% total errors
DFA	1032	111	8	103	10
DNA	630	58	1	57	9
CON	273	7	1	6	2
AA	9911	1018	224	794	10
CC	175	45	34	11	25
CD	516	129	103	26	25
CE	5453	223	204	19	4
CQ	28	7	3	4	25
AE	1765	66	58	8	3
SE	940	33	29	4	3
DIFF	0	0	0	0	0
DC	1034	185	29	156	17
DD	2363	333	56	277	14
DM	1951	134	19	115	6
DN	1610	40	40	0	2
DS	13	8	8	0	61
FC	3081	157	157	0	5
FD	1516	62	62	0	4
HS	314	29	4	25	9
LB	152	23	6	17	15
LC	4742	163	31	132	3
LT	4602	172	160	12	3
RC	4550	186	40	146	4
QNA	232	45	2	43	19
QNB	816	36	10	26	4
QNC	258	15	4	11	5
QND	979	26	5	21	2
CA	2745	155	80	75	5
AS	619	67	20	47	10
Z	1352	113	25	88	8
N	1228	108	20	88	8
AI	1033	91	16	75	8
U	251	11	11	0	4
NS	552	14	11	3	2
AD	37	1	1	0	2
TR	163	8	5	3	4
CN	124	6	1	5	4
DG	138	13	8	5	9
NL	734	13	12	1	1
NG	561	10	9	1	1
AR	1233	207	26	181	16
AL	677	142	8	134	20

(Table XIIIb, cont.)

Rule	No. of times rule used	No. of times error in use	Syntax errors	Appl. errors	% total errors
UI	713	60	11	49	8
CU	1030	44	10	34	4
DI	762	71	25	46	9
DU	612	65	20	45	10
II	491	34	9	25	5
RA	679	38	9	29	5
EM	650	36	8	28	5
UA	15	6	0	6	40
UR	210	32	8	24	15
UL	115	34	3	31	29
SA	143	10	6	4	6
CS	154	7	7	0	4
IA	230	39	6	33	16
TH1	528	40	21	19	7
TH2	575	52	15	37	9
TH3	192	9	6	3	4
TH4	177	12	2	10	6
TH5	198	7	7	0	3
TH6	94	3	1	2	3
TH7	58	15	4	11	25
TH8	5	0	0	0	0
TH9	1	0	0	0	0
TH10	0	0	0	0	0
TH11	1	0	0	0	0
TH12	0	0	0	0	0
TH13	5	0	0	0	0
TH14	0	0	0	0	0
TH15	0	0	0	0	0
TH16	0	0	0	0	0
TH17	1	0	0	0	0
TH18	0	0	0	0	0
TH19	0	0	0	0	0
TH20	1	0	0	0	0
TH21	2	1	1	0	50
TH22	2	0	0	0	0
TH60	1	0	0	0	0
TH61	165	0	0	0	0
TH62	212	6	6	0	2
TH63	4	1	1	0	25
TH64	1	0	0	0	0
TH65	3	0	0	0	0
TH66	3	1	1	0	33
TH67	0	0	0	0	0
TH68	2	0	0	0	0
TH69	7	0	0	0	0

(Table XIIIb, cont.)

Rule	No. of times rule used	No. of times error in use	Syntax errors	Appl. errors	% total errors
TH70	99	1	1	0	1
TH71	53	1	1	0	1
TH72	54	0	0	0	0
TH161	164	6	3	3	3
TH162	93	2	1	1	2
TH163	167	10	1	9	5
TH164	230	6	2	4	2
TH165	204	14	5	9	6
TH166	291	11	1	10	3
TH167	158	15	7	8	9
TH168	268	18	5	13	6
TH169	16	5	4	1	31
TH170	10	0	0	0	0
TH171	9	0	0	0	0
TH172	4	2	2	0	50
TH173	72	18	15	3	25
TH174	130	7	6	1	5
TH175	2	0	0	0	0
TH176	2	0	0	0	0
TH177	46	1	0	1	2
TH178	29	0	0	0	0
TH179	1	0	0	0	0
TH180	63	1	0	1	1
TH181	42	7	1	6	16
TH182	5	0	0	0	0
TH183	1	0	0	0	0
TH184	2	0	0	0	0
TH185	1	0	0	0	0
TH186	3	1	0	1	33
TH187	89	6	6	0	6
TH188	38	6	5	1	15
TH189	19	6	6	0	31
TH190	18	2	2	0	11
TH191	16	0	0	0	0
TH192	3	1	1	0	33
TH193	0	0	0	0	0
CP	12729	1660	447	1213	13
DLL	8251	110	83	27	1
EG	2368	249	6	243	10
ES	4715	1138	84	1054	24
HINT	2449	1324	0	1324	54
HYP	9730	0	0	0	0
IP	4917	788	174	614	16
ND	2730	98	93	5	3

(Table XIIIb, cont.)

Rule	No. of times rule used	No. of times error in use	Syntax errors	Appl. errors	% total errors
RE	8836	784	183	601	8
REVIEW	2728	0	0	0	0
RQ	122	50	35	15	40
UG	3491	254	61	193	7
US	5492	406	70	336	7
WP	13626	23	23	0	0

the next section.) The percentage of errors in frequently used sentential rules was low except for IP (indirect proof procedure). The IP errors are application errors, usually seen as the inability to select the two lines containing formulas that negate one another, or, if properly selected, to order the three line references. Undoubtedly, the percentage of errors for IP would decrease if the program were slightly less strict on the order of line references (unless order were essential, as it is in forming a conjunction). This is a simple programming change.

Rules for commuting a disjunction or conjunction (CD and CC) also showed high error rates. If the difficulty were in determining whether a formula is a conjunction, disjunction, or implication, the error would show as an application error. Rather they are syntax errors, possibly omitting or incorrectly referring to the occurrence of the logical connective.

Application errors in using commands that regroup parentheses (AS, AR, AL, UR, UL, UR) are also exceptionally high. These are difficult rules to use because they demand recognition of patterns or axiom schemata in which grouping is important, and require the ability to count occurrences (in a left-to-right manner) of these patterns in a formula. AR and AL are short forms for AS; similarly, UL and UR are short forms for US. Since the number of errors in using short forms of other axioms (Table XIV) is not as high, and those other axioms have simpler structures,

Insert Table XIV about here

we can conclude that extra depth of parenthetical structure is a source of difficulty.

TABLE XIVa

Data on Use of Short Forms of Rules Summed over Students and Exercises (Spring, 1973)

Rule	No. of times used	No. as short forms	Error in short forms
CA	3165	2421	7
AS	650	44	0
Z	1635	728	9
N	1490	939	4
AI	1195	505	8
U	113	0	0
NS	564	140	0
AD	8	1	0
TR	164	14	0
CN	134	16	0
DG	116	51	0
NL	567	0	0
NG	472	0	0
UI	774	262	2
CU	985	937	12
DI	821	406	10
DU	699	449	9
II	543	280	4
RA	740	203	3
EM	683	235	7
UA	27	12	0
SA	178	39	0
CS	161	71	3
IA	241	42	0
TH1	580	201	3
TH2	603	274	0
TH3	293	120	1
TH4	195	50	0
TH5	254	105	0
TH6	98	22	0
TH7	60	35	0
TH8	2	0	0
TH9	1	0	0
TH13	1	0	0
TH14	1	0	0
TH16	3	0	0
TH17	1	0	0
TH21	2	0	0
TH22	1	0	0

(Table XIVa, cont.)

Rule	No. of times used	No. as short forms	Error in short forms
TH60	10	0	0
TH61	161	0	0
TH62	148	43	0
TH63	2	0	0
TH64	1	0	0
TH65	1	0	0
TH66	5	0	0
TH67	4	0	0
TH69	2	0	0
TH70	77	0	0
TH71	55	0	0
TH72	61	0	0
TH161	96	15	0
TH162	79	4	0
TH163	199	96	0
TH164	266	152	4
TH165	228	81	6
TH166	273	92	3
TH167	153	36	0
TH168	199	30	0
TH169	16	2	0
TH170	18	2	0
TH171	13	0	0
TH172	7	0	0
TH173	78	12	0
TH174	122	8	0
TH175	7	1	0
TH176	7	0	0
TH177	36	24	0
TH178	37	9	0
TH179	21	1	0
TH180	45	19	0
TH181	33	7	0
TH182	13	6	0
TH183	5	0	0
TH184	1	0	0
TH185	2	0	0
TH186	1	0	0
TH187	72	34	2
TH188	45	10	0
TH189	7	1	0
TH190	12	4	0
TH191	10	3	0
TH192	3	0	0
DLL	2017 (as restart)		

TABLE XIVb

Data on Use of Short Forms of Rules Summed over Students and Exercises (Fall, 1973)

Rule	No. of times used	No. as short forms	Error in short forms
CA	2745	2054	20
AS	619	65	0
Z	1352	576	3
N	1228	663	4
AI	1033	406	2
U	251	0	0
NS	552	57	1
AD	37	0	0
TR	163	7	0
CN	124	12	0
DG	138	16	0
NL	734	0	0
NG	561	0	0
UI	713	304	0
CU	1030	939	10
DI	762	311	2
DU	612	274	1
II	591	289	1
RA	679	210	0
EM	650	251	1
UA	15	1	0
SA	143	11	0
CS	154	37	0
IA	230	40	0
TH1	528	194	2
TH2	575	269	5
TH3	192	63	0
TH4	177	39	0
TH5	198	53	0
TH6	94	37	2
TH7	58	23	0
TH8	5	0	0
TH9	1	0	0
TH11	1	0	0
TH13	5	1	0
TH17	1	0	0
TH20	1	1	0
TH21	2	0	0
TH22	2	0	0

(Table XIVb, cont.)

Rule	No. of times used	No. as short forms	Error in short forms
TH60	1	0	0
TH61	165	1	0
TH62	212	21	0
TH63	4	0	0
TH64	1	0	0
TH65	3	0	0
TH66	3	0	0
TH68	2	0	0
TH69	7	0	0
TH70	99	0	0
TH71	53	0	0
TH72	54	0	0
TH161	164	21	0
TH162	93	1	0
TH163	167	74	2
TH164	230	138	2
TH165	204	75	0
TH166	291	122	0
TH167	158	51	0
TH168	268	24	0
TH169	16	0	0
TH170	10	0	0
TH171	9	0	0
TH172	4	0	0
TH173	72	4	0
TH174	140	6	0
TH175	2	0	0
TH176	2	0	0
TH177	46	33	0
TH178	29	6	0
TH179	1	0	0
TH180	63	22	0
TH181	42	9	0
TH182	5	4	0
TH183	1	0	0
TH184	2	0	0
TH185	1	0	0
TH186	3	0	0
TH187	89	18	0
TH188	38	10	0
TH189	19	5	0
TH190	18	6	0
TH191	16	8	0
TH192	3	0	0
DLL	2601 (as restart)		

Most theorems, if used frequently, do not have high error rates. The ones the students did use frequently were those useful in proving that their interpretations of arguments (Lesson 428) were correct: TH61 and TH62. Of these, only TH61 shows a noticeable error rate in the spring class--mainly syntax errors. The students' attempts to use the short forms of the theorems met with considerable success in terms of the number of errors.

Of the quantifier rules, students demonstrated the most difficulty in learning existential specification (ES), as is clearly seen from the data in Table XIII. The errors were mostly application errors--selecting an ambiguous name that is either not well formed or is already introduced in the proof. The errors in UG, US, and EG were also application errors--selecting a term that cannot be used as a variable of generalization or attempting to specify a term such that the term contains a free occurrence of a variable that will be captured by a quantifier using that variable. Application errors in using the first quantifier negation rule were also high. This is probably due to some confusion in determining the scope of a quantifier and of a negation symbol. Profiles of individual error histories were also constructed, but are not included for reasons of space.

Student-defined Rules

One of the commands the student can use is INIT, a request to select a different problem from the curriculum or to make up his own problems. The problem the student invents can be a derivation (DERIVE command) to test out his own notions about what constitutes a problem, or a proof

(PROVE command) to prove a lemma that can help in completing subsequent curriculum problems. Using PROVE, the student constructs a proof of some well-formed formula and then provides a name, a label with which he later refers to the formula.

Three significant statements can be made from these data.

(1) Students did take advantage of INIT mode (an average of approximately 21 times in the spring and 22 in the fall), and they did use this mode to extend their command language for constructing proofs. Twenty-one out of 38 fall students and 23 out of 41 spring students completed lemmas. Of these, 14 fall and 15 spring students received grades of A. Every student used his lemmas at least once in subsequent derivation problems. An average number of 3.4 (3.9 in the fall) lemmas were proved; these were used an average number of 17.5 times (28.7 for fall students).

(2) The students, without exception, never made an error in using a lemma (command) they proved.

(3) The names of the lemmas, except for a few cases (students 2850, 2856, 2857, 2861, and 2867--all spring students), are all nonsense names. We find anything from BANANA to PREHISTORICMECHANICALBEAST, from ALLDONE-WITHFINDINGAXIOMS to WOWIFINISHEDIT, and from swear words to names of politicians. The semantic significance of this use of nonsense names certainly needs further investigation. The same use of nonsense words was prevalent in the finding-axioms exercises in naming axioms and theorems.

Not all the students, of course, used INIT mode to make up problems. But those that did used INIT to

(1) try out the suggested proofs from lesson 418, e.g.,

B+C=D → B=D-C

B=D-C → B+C=D

B+C=0 → B=-C ;

(2) make up lemmas to help in completing interpretation problems, e.g.,

(E X) (X=X & X<5)

NOT (A X)(X=X → NOT X=X) ;

(3) prove new formulas from Boolean algebra.

Experimentation with the proof-checking program was, however, minimal in contrast to the program's actual design goals. The course is non-trivial; it takes time to complete all 33 lessons and 7 finding-axioms exercises. Because course grades are assigned according to the number of lessons completed, the students were anxious to complete the assigned curriculum and hesitated to spend time on extracurricular problems.

Two Student Profiles

In this section we attempt to identify characteristics of the students' work that reflect salient individual differences. We do this by analyzing the profiles of two students.

Both students were in the spring class and both received grades of A. That is where the similarity ends. One was the first student to complete the course, the other almost the last. One always completed lessons faster than the average time, the other always slower. One spent time making up his own problems (and helping debug the curriculum), the other stayed within the framework of stored curriculum. One worked afternoons, the other late at night.

More systematically, we examined the data records of the two students for the following features. (We have also commented parenthetically how information about each feature might be used to improve the course.)

1. Error rate in using rules. (This information is needed for modifying the choice of problems.)

2. Frequency of syntax or application errors. (Application errors can be further analyzed in terms of knowledge of kinds of well-formed formulas, pattern recognition, or understanding of the restrictions of the rule.)

3. Number of steps in completed solutions relative to the average. (If student fails to see and use rules or methods for shortening his derivations, some additional discussion might be appropriate.)

4. Use of rules of inference. (Does he become confused if presented with a problem not requiring recently proved or learned rules?)

5. Time to complete lessons and deviation from the average. (Does he need more or less practice, or should time be ignored completely in favor of error rates?)

6. Request for hints. (Is the student seeking a quick path through the curriculum and looking for answers?)

7. Use of INIT mode. (Does this usage affect the length of time spent on a lesson?)

8. Proof of lemmas. (Does student use them to help solve the more difficult exercises?)

9. Data on choice of derive, truth analysis, or interpretation mode. (Does student need more such choice exercises to develop his intuition about logical validity?)

In terms of the above features, the profiles of the two students we have selected are distinct, except for the number of hints requested. The fact that the student who was performing so well requested so many hints confirms our observation that he was interested in 'racing through' the curriculum for credit because his strong mathematics background included prior study of the material. We label the students 'A' and 'B'; Table XV highlights the differences in their performance.

Insert Table XV about here

Student A encountered several curriculum errors because he was usually the first student to reach a lesson. This probably accounts for the high number of sessions and the need to use the LESSON command to skip over problems that were incorrectly stated. He still spent less than the average amount of time, doing lessons in less time than the average, and making few errors in rule usage. Rule ES (existential specification) gave him trouble, but still less than it gave other students. His intuition on doing DI problems was comparatively poorer than average. He did make good use of INIT mode, especially for interpretation and Boolean algebra problems. He was, in fact, the only one to use INIT to prove lemmas in the Boolean algebra. He was not necessarily interested in finding minimum proofs, doing about the average number of steps. And, finally, he had to learn only once that he would have to follow the constraints set in the curriculum for each problem. This computer-based course suited Student A because he could finish early and concentrate on other courses.

TABLE XV

Profiles of Two Students

Feature	Student A	Student B
Grade	A	Incomplete; finished with A grade the next quarter
Progress	Finished first	Finished with A grade the next quarter
Number of sessions	96 (above average)	288 (excessive)
Time spent	41.5 hours (below average)	111.8 (excessive)
Deviation from average	Always faster than average	Usually slower
Work hours	4:00 p.m. to 5:00 p.m. preferred	Nights, 8:00 p.m. or 9:00 p.m., 11:00 p.m. to midnight
Rule usage	Generally better than average	Except for AR, always worse than average
% errors for (Average % errors)		
AR (17)	4	12
CP (9)	0	19
IP (17)	0	49
UG (6)	7	20
US (6)	5	13
EG (9)	7	15
ES (25)	14	42
Choice problems (% correct on 1st or 2nd try)		
DC	(Data lost due to system crash)	55%
DI	57%	100%
DIC	100%	100%
Hints	70 hints requested; 62% not available	65 hints requested; 61% not available

(Table XV, cont.)

Feature	Student A	Student B
Short forms of axioms and theorems	Average amount for both axioms and theorems	Only used for axioms
INIT mode times types (21 average)		
No. of DERIVEs	70	30
No. of PROVEs	0	0
No. PROVEs completed (3 average)	20	0
No. times Lemmas used	36	0
No. times short form used	11	0
No. of steps to solution	Average or below average	Fluctuated a lot, often finding minimum or average proofs but as often doing the maximum
LESSON command times requested (11 average)	49	0
REVIEW (43 average)	28	154
REDO total no. times had to redo problem	1	8

Student B was a slow reader, taking a long time to do even the first two lessons. He experienced a lot of trouble doing the lessons on interpretation and quantifier rules and always spent more time than the other students on each lesson. He spent over 100 hours on the course, more than double the average, working mainly at night (when a teaching assistant was, unfortunately, not available to help him). All the rules gave him some trouble; IP and ES errors were application errors. We include his cumulative error curve for IP as an example (Figure 18); its approximate linearity indicates little improvement from additional use of the rule. He never ventured to complete his own lemmas and never deviated

----
Insert Figure 18 about here
----

from the assigned, linear sequence of problems (despite the fact that redoing some problems might have given him needed reviews).

Although the course was often a frustrating experience for Student B, he would almost certainly have received a low grade in a nonindividualized, non-computer-based course. He had the opportunity to interact with and complete every problem. It took some prodding to convince him to continue; but he did, receiving a deserved grade of A.

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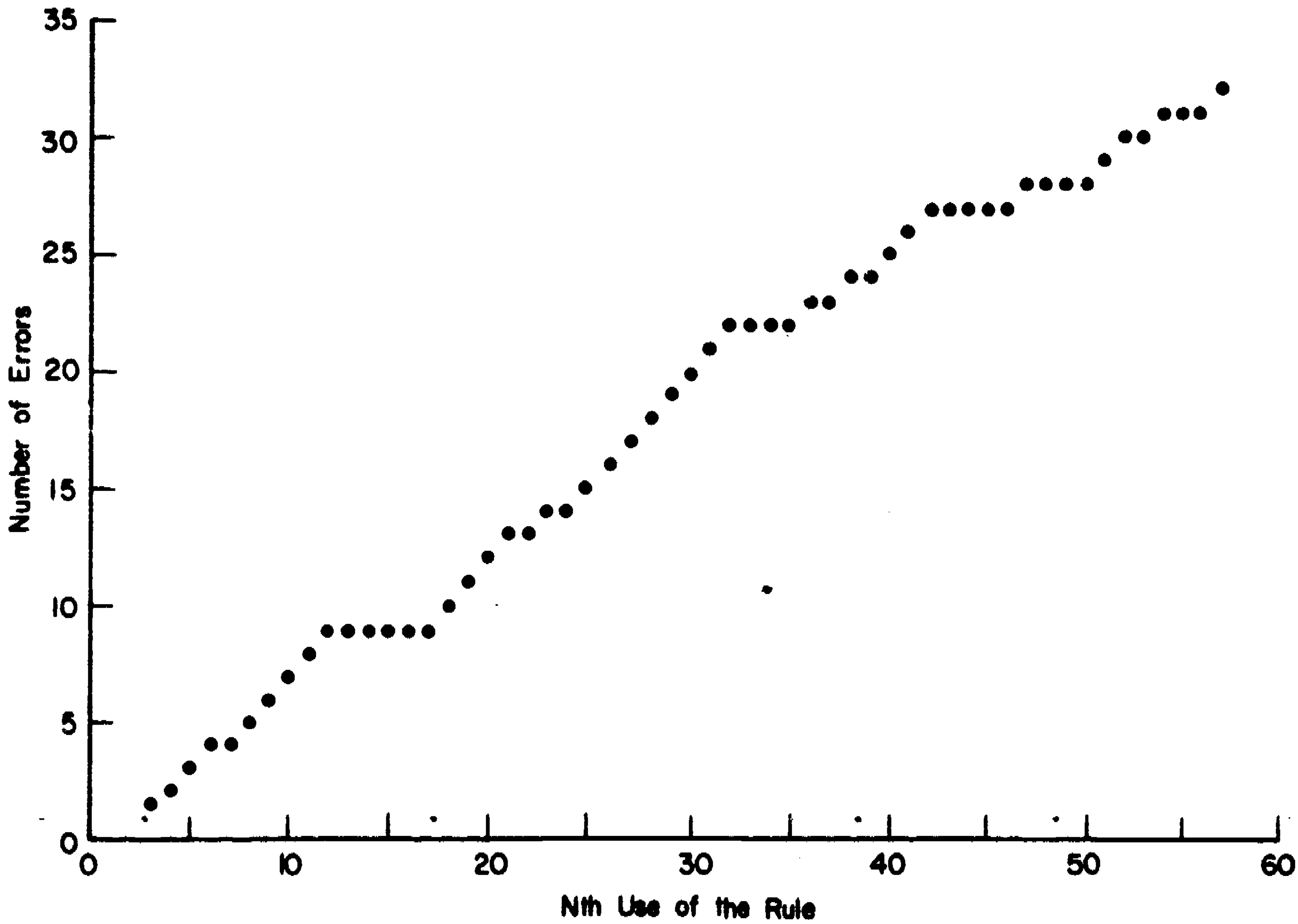

Fig. 18. Distribution of errors in use of rule IP by Student B.

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FOOTNOTE

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APPENDIX I

Student Evaluation Form for Computer-based Course in Symbolic Logic

I. Some factual information.

1. What grade did you receive in the course?
2. What grade did you expect to receive when you started the course?
3. How did you find out about the course?
4. Did you find having the INIT rule useful? If not, why not?
5. Were you always aware that HINT was a legal command in constructing proofs?
6. Do you think the problems need more hints?
7. Did you attend the Tuesday seminars? If not, why not?

II. Problem types.

1 Derivations and proofs
2 Truth analysis
3 Counterexamples using truth analysis
4 Derive or give a counterexample
5 Counterexample by interpretation in the domain of rational numbers
6 Derive to show argument valid or give an interpretation to show invalid
7 Interpretation to show premises consistent
8 Derive to show premises inconsistent or give interpretation to show consistent
9 Interpretation to show axioms independent
10 Derive to show axioms dependent or give interpretation to show independent
11 Translate English sentences into first-order logic--derive to show answers logically equivalent ("A" people only)
12 Finding-axioms exercises

1. Which problem type was the easiest?
2. Which problem type was the hardest?
3. Which problem types did you like best?
4. Which problem types did you like least?
5. Which specific problems, if any, did you think were too hard?

III. Please read each statement and circle the number on the scale that best describes your feelings.

Scale

1 Strongly agree
2 Moderately agree
3 Slightly agree
4 Uncertain
5 Slightly disagree
6 Moderately disagree
7 Strongly disagree

1.	I think I learned from the computer lessons as well as I would have learned the same lessons in the classroom.	1 2 3 4 5 6 7
2.	I like working at my own pace at the terminal.	1 2 3 4 5 6 7
3.	I prefer homework to working on problems at the terminal.	1 2 3 4 5 6 7
4.	I would prefer competing with my fellow students in the classroom rather than working at computer lessons.	1 2 3 4 5 6 7
5.	I find it frustrating not knowing where my fellow classmates are in the lessons.	1 2 3 4 5 6 7
6.	Working with computer lessons is like having my own tutor.	1 2 3 4 5 6 7
7.	Five hours a week is sufficient time to keep up with the course.	1 2 3 4 5 6 7
8.	I found the computer lessons too easy.	1 2 3 4 5 6 7
9.	I think working with computer lessons is an exciting way to learn.	1 2 3 4 5 6 7
10.	I found working at the terminal frustrating.	1 2 3 4 5 6 7
11.	I would like to participate in another computer-based course.	1 2 3 4 5 6 7
12.	I found the computer lessons too hard.	1 2 3 4 5 6 7
13.	The computer provides the student with more feedback than classroom instruction.	1 2 3 4 5 6 7

14. The terminals were available to me when I wanted to work. 1 2 3 4 5 6 7

15. There was sufficient outside help when I needed it. 1 2 3 4 5 6 7

16. Use the back of this sheet to make any comments you wish concerning the course.

APPENDIX [illegible]

Minimum and Maximum Number of Steps for All Derivation Problems

Lesson	Problem	Random choice	Number of steps: Min	Number of steps: Max
403	9	0	3	3
	10	0	3	3
	13	0	3	3
	14	1	4	4
	14	[illegible]	4	[illegible]
	15	1	4	4
	1[illegible]	[illegible]	4	4
	1[illegible]	1	5	[illegible]
	1[illegible]	[illegible]	5	5
	[illegible]	1	7	7
	[illegible]	2	[illegible]	7
	18	1	5	5
	18	2	[illegible]	[illegible]
404	1	0	2	3
	[illegible]	0	3	4
	[illegible]	0	2	4
	1[illegible]	1	3	10
	11	2	3	11
	1[illegible]	1	[illegible]	11
	1[illegible]	2	3	5
	13	1	[illegible]	9
	13	2	[illegible]	7
	[illegible]	1	[illegible]	[illegible]
	18	[illegible]	8	8
	[illegible]	1	5	12
	[illegible]	1	6	8
	22	2	6	6
405	2	1	2	10
	2	2	[illegible]	10
	3	1	3	10
	3	2	3	9
	4	1	4	17
	4	2	7	16
	[illegible]	1	3	16
	5	2	3	8
	7	1	2	10
	7	2	2	8
	8	1	5	17
	8	2	3	12
	9	1	7	11
	9	2	2	11
405	10	1	8	15
	10	2	7	8
	11	1	8	11
	11	2	5	10
	12	1	8	10
	12	2	10	15
	13	1	3	8
	13	[illegible]	7	8
	14	[illegible]	6	6
	14	[illegible]	6	6
406	2	1	3	3
	2		3	3
	3	1	3	6
	3	2	3	3
	4	1	3	6
	4	2	3	3
	5	1	6	6
	5	2	6	6
	7	1	3	5
	7	2	3	3
	8	1	3	5
	8	2	3	3
	9	1	5	5
	9	2	5	6
	10	1	5	5
	10	2	5	6
	12	1	2	6
	12	2	2	3
	13	1	5	8
	14	1	4	10
	14	2	6	6
	15	1	5	8
	15	2	5	7
	16	1	8	11
	16	2	6	8
	17	1	8	9
	17	2	6	9
	18	1	6	9
	18	2	6	9
	19	1	6	22
	19	2	7	12

Lesson	Problem	Random choice	Number of steps	
			Min	Max
406	20	1	9	9
	20	2	9	10
	21	1	8	11
	21	2	11	12
	22	1	8	12
	22	2	8	9
	23	1	3	5
	23	2	4	6
	24	1	3	4
	24	2	4	5
407	1	0	3	6
	3	1	3	4
	3	2	3	3
	4	1	5	8
	4	2	3	8
	5	1	5	6
	5	2	6	6
	6	1	3	5
	6	2	4	4
	7	0	5	5
	11	1	2	7
	11	2	5	6
	12	1	5	6
	12	2	5	6
	13	1	5	6
	13	2	5	5
	14	1	5	8
	14	2	6	10
	15	1	5	5
	15	2	4	6
	16	1	5	6
	16	2	4	6
	19	0	3	4
	20	1	3	5
	20	2	3	4
	21	1	3	7
	21	2	2	9
408	6	0	3	9
	7	1	4	9
	7	2	7	8
	8	1	6	11
	8	2	6	10
	9	1	4	7
	9	2	3	6
	10	1	3	11
408	10	2	8	17
	11	1	6	10
	11	2	6	9
	12	1	8	10
	12	2	6	11
	13	1	4	7
	13	2	4	5
	14	1	7	8
	14	2	5	10
	15	1	6	18
	15	2	8	9
	16	1	7	9
	16	2	7	18
	17	1	7	8
	17	2	7	9
	18	1	7	9
	18	2	7	8
	19	1	5	7
	19	2	5	7
	20	0	7	8
	21	0	7	7
	22	0	10	15
	23	1	10	12
	23	2	12	14
	24	1	3	8
	24	2	6	9
	25	1	8	14
	25	2	8	14
	26	1	5	10
	26	2	5	10
	27	0	13	17
	28	0	15	20
409	1	0	4	8
	12	1	3	10
	13	1	5	10
	16	2	7	12
	18	0	5	8
	20	1	8	13
	22	0	5	7
	24	1	4	14
	24	2	10	17
	25	1	5	18
	25	2	8	12
	26	1	9	15
	26	2	8	19

Lesson	Problem	Random choice	Number of steps	
			Min	Max
410	16	1	5	7
	16	2	5	9
	17	1	3	5
	17	2	3	5
	18	1	3	5
	18	2	5	7
	22	1	1	3
	22	2	1	1
	23	1	1	1
	23	2	1	1
	24	1	3	4
	24	2	3	5
	25	1	5	7
	25	2	5	7
	26	1	7	9
	26	2	4	7
	27	1	4	8
	27	2	4	9
	28	1	5	6
	28	2	4	7
	29	1	6	8
	29	2	7	9
411	9	1	2	8
	9	2	3	4
	10	1	2	3
	10	2	2	2
	11	1	3	6
	11	2	7	11
	12	1	5	6
	12	2	6	8
	13	1	9	10
	13	2	4	10
	14	1	3	3
	14	2	4	17
	20	1	2	2
	20	2	3	3
	21	1	2	2
	21	2	2	4
	23	1	2	3
	23	2	2	2
	24	1	3	3
	24	2	4	4
	26	1	3	3
	26	2	4	5
	27	1	3	5
411	27	2	3	4
	28	1	4	4
	28	2	4	[illegible]
412	4	1	2	2
	4	2	2	2
	5	1	2	3
	5	2	2	3
	6	1	2	4
	6	2	3	4
	7	1	3	3
	7	2	3	4
	8	1	3	5
	8	2	3	5
	9	1	3	3
	9	2	5	6
	10	1	3	8
	10	2	3	4
	11	1	3	3
	12	1	3	5
	12	2	4	4
	13	1	8	12
	13	2	7	7
	20	1	1	2
	20	2	1	4
	21	1	6	6
	21	2	6	8
	22	1	5	6
	22	2	6	8
	23	1	3	4
	23	2	3	3
	24	1	8	10
	24	2	9	10
	25	1	3	4
	25	2	5	6
	26	1	1	5
	26	2	1	4
	27	1	5	7
	27	2	7	9
	28	1	1	2
	28	2	4	5
	29	1	4	5
	29	2	4	6
	30	1	5	8
	30	2	3	6

Lesson	Problem	Random choice	Number of steps	
			Min	Max
413	23	0	4	7
414	1	0	3	[illegible]
	2	1	3	[illegible]
	2	2	4	9
	4	1	3	4
	4	2	6	9
	[illegible]	1	4	7
	[illegible]	2	4	7
	6	1	3	5
	[illegible]	2	3	6
	7	1	7	11
	7	2	8	14
	8	1	5	8
	8	2	6	8
	11	1	3	6
	11	2	5	8
	12	1	12	19
	13	1	9	14
	13	2	10	16
	14	1	5	5
	14	2	5	9
	15	1	[illegible]	8
	15	2	8	12
	16	1	7	9
	16	2	6	8
	17	1	8	14
	17	2	6	12
	18	1	5	5
	18	2	6	7
	19	1	6	9
	19	2	3	10
	20	1	6	12
	20	2	7	10
415	21	1	1	2
	21	2	1	5
	22	1	1	2
	22	2	1	2
	23	1	3	6
	23	2	3	5
	24	1	1	4
	24	2	3	6
	25	1	1	9
	25	2	1	11
	26	1	3	9
	26	2	3	9
415	27	1	2	2
	27	2	2	4
	28	1	2	2
	28	2	2	4
	29	1	3	4
	29	2	4	7
	30	1	2	3
	30	2	2	5
	31	1	5	11
	31	2	1	5
	32	1	10	10
	32	2	9	15
416	4	0	1	3
	6	1	1	4
	6	2	2	11
	7	1	5	9
	7	2	4	7
	8	1	6	12
	8	2	8	17
	9	1	3	13
	9	2	5	15
	14	0	2	2
	15	1	2	2
	15	2	4	6
	16	1	2	4
	16	2	2	2
	17	1	3	4
	17	2	3	5
	18	1	4	6
	18	2	5	7
	19	1	11	19
	19	2	11	17
	20	1	7	14
	20	2	7	14
	21	1	4	9
	21	2	4	9
	22	1	8	21
	22	2	9	25
	23	1	7	10
	23	2	7	10
417	4	0	1	2
	5	1	1	2
	5	2	1	5
	6	1	1	5
	6	2	1	4

Lesson	Problem	Random choice	Number of steps	
			Min	Max
417	7	1	3	7
	7	2	3	10
	9	1	5	7
	9	2	3	5
	10	1	2	3
	10	2	2	4
	15	1	1	5
	15	2	1	3
	16	1	2	8
	16	2	2	6
	17	1	4	15
	17	2	8	21
	18	1	5	11
	18	2	7	25
	19	1	7	15
	19	2	7	12
	22	1	1	1
	22	2	2	2
	23	1	5	16
	23	2	5	13
	24	1	11	20
	24	2	4	10
	25	1	2	5
	25	2	2	5
	26	1	3	4
	26	2	4	6
	27	1	7	13
	27	2	5	7
	28	1	2	5
	28	2	5	8
	29	1	3	11
	29	2	5	9
418	3	0	2	7
	5	0	1	2
	6	1	1	1
	6	2	1	2
	8	1	4	34
	8	2	7	15
	10	1	2	2
	10	2	2	5
	12	1	5	16
	12	2	6	15
	13	0	1	4
	14	1	3	16
	14	2	7	28

Lesson	Problem	Random choice	Number of steps	
			Min	Max
418	15	0	2	4
	16	1	2	5
	16	2	3	7
	17	1	7	20
	17	2	3	8
	18	1	3	7
	18	2	10	22
	19	1	4	7
	19	2	5	7
	21	1	3	6
	21	2	4	6
	25	1	5	24
	25	2	4	25
	27	1	12	18
419	2	1	2	3
	2	2	2	4
	3	1	2	6
	3	2	3	12
	4	1	3	9
	4	2	3	11
	6	1	3	12
	6	2	5	10
	7	0	3	5
	8	1	2	11
	8	2	5	9
	9	1	2	11
	9	2	5	23
	10	0	4	6
	11	1	3	9
	11	2	3	8
	12	1	2	12
	12	2	3	15
	13	1	3	13
	13	2	4	18
	14	0	11	22
	15	1	4	15
	15	2	8	26
	17	1	4	19
	17	2	2	21
	18	1	5	11
	18	2	3	14
	19	1	7	11
	19	2	5	16
420	4	0	7	34
	5	0	4	21

Lesson	Problem	Random choice	Number of steps Min	Max
420	6	0	2	9
	7	0	2	15
	8	0	2	12
	9	0	3	24
	10	0	10	27
422	19	0	4	7
	23	0	7	11
	27	0	3	14
	28	0	4	23
	35	0	14	27
	39	0	7	20
424	10	0	2	5
	11	0	3	8
	12	0	4	10
426	3	0	2	3
	4	0	12	18
	6	0	6	10
	7	0	6	9
	8	1	6	10
	8	2	6	8
	9	1	4	16
	9	2	7	11
	10	1	6	13
	10	2	4	6
	11	1	6	25
	11	2	11	19
	12	1	3	6
	15	0	7	11
	16	1	4	6
	16	2	5	10
	19	0	5	7
	24	0	2	2
	25	0	2	3
	26	0	2	4
	27	0	6	7
	28	1	9	11
	28	2	9	13
	31	1	7	15
	31	2	9	10
	32	1	11	14
	32	2	7	11
	36	0	9	14
	37	1	9	13
	37	2	9	11

Lesson	Problem	Random choice	Number of steps Min	Max
426	38	1	9	25
	38	2	12	29
	40	1	9	11
	40	2	10	17
	44	0	7	17
427	8	0	2	5
	31	0	2	7
	32	1	3	24
	32	2	11	25
	33	1	7	20
	33	2	11	20
	34	1	4	6
	34	2	2	4
	35	0	6	6
	36	0	5	19
	37	0	5	6
	38	0	5	6
	39	0	5	6
	40	0	4	7
	41	0	4	5
	42	0	4	7
	43	0	8	25
	44	0	9	14
	45	0	5	9
	46	0	5	8
	49	0	8	18
428	13	0	3	9
	14	0	3	5
	15	0	3	3
	16	0	5	10
	17	0	3	12
	18	1	3	6
	18	2	4	9
	19	1	3	10
	19	2	4	10
	20	0	7	14
	21	1	11	17
	21	2	11	20
	22	1	7	18
	22	2	7	13
	23	1	4	16
	23	2	6	14
	24	0	24	29
	25	0	3	6
	26	0	6	12

Les-son	Prob-lem	Random choice	Number of steps Min	Max
429	6	0	14	14
	10	0	3	8
	11	0	8	18
	12	0	3	19
	13	0	3	26
	14	0	2	15
	17	0	4	4
	18	0	3	6
	20	0	7	19
	21	0	8	25
	22	0	4	19
431	2	0	5	10
	3	0	7	9
	7	0	4	15
	9	0	1	3
	10	0	1	3
	14	0	6	11
	[illegible]	0	2	7
	[illegible]	0	7	16
	[illegible]	0	1	13
	[illegible]	0	7	24
	[illegible]	0	2	13
	[illegible]	0	7	15
	[illegible]	0	9	26
	[illegible]	0	6	8
	[illegible]	0	2	10
	33	0	11	18
	34	0	4	53
[illegible]	8	0	3	12
	9	0	3	9
	10	0	9	35
	11	0	8	29
	12	0	5	20
	13	0	5	15
	14	0	4	23

Les-son	Prob-lem	Random choice	Number of steps Min	Max
432	15	0	4	9
	16	0	5	[illegible]
	17	0	[illegible]	[illegible]
	21	0	4	33
	24	0	6	22
	25	0	5	12
	27	0	10	35
	28	0	6	29
	30	0	17	51
	33	0	15	37
	34	0	14	33
	37	0	1	26
	38	0	3	20
	41	0	6	15
	42	0	5	14
	43	0	5	14
	44	0	14	38
	45	0	10	28
	46	0	8	39
	47	0	26	95
433	5	0	2	5
	6	0	3	12
	7	0	4	24
	8	0	24	57
	9	0	9	30
	10	0	11	30
	12	0	6	63
	13	0	7	37
	14	0	11	46
435	11	0	3	6
	12	0	3	8
	13	0	3	19
	16	0	7	32
	17	0	7	21

APPENDIX III

Logical Rules of Inference and Proof Procedures

(In the following, i, j, and k are variables for line numbers.)

1) Affirm the antecedent

	i) P → Q
	j) P
i,jAA	k) Q

2) Commute conjuncts

	i) P & Q
iCC1	j) Q & P

3) Commute disjuncts

	i) P OR Q
iCD1	j) Q OR P

4) Commute equals

	i) B = C
iCE1	j) C = B

5) Commute equivalence

	i) Q IFF R
iCQ1	j) R IFF Q

6) Conditional proof procedure

WP	i) R	line i must be the
	j) Q	last working premise
i,jCP	k) R → Q	introduced

7) Contrapositive

	i) R → Q
iCON	j) (NOT Q) → (NOT R)

8) Deny the consequent

	i) P → R	i) P → NOT R
	j) NOT R	j) R
i.jDC	k) NOT P	k) NOT P

9) Deny a disjunct

	i) R OR Q	i) R OR Q
	j) NOT R	j) NOT Q
i.jDD	k) Q	k) R

10) Definition of implication

	i) Q → R	i) (NOT Q) OR R
iDFA	j) (NOT Q) OR R	j) Q → R

11) De Morgan's Law

	i) NOT (R OR Q)	i) NOT (R & Q)
iDM	j) NOT R & NOT Q	j) NOT R OR NOT Q

12) Double negation

	i) R	i) NOT (NOT R)
iDN	j) NOT (NOT R)	j) R

13) Distribute negation over implication

	i) NOT (Q → R)	i) Q & NOT R
iDNA	j) Q & NOT R	j) NOT (Q → R)

14) Disjunctive syllogism

	i) Q OR R
	j) Q → P
	k) R → W
i.j.kDS	
	m) P OR W

15) Existential generalization

	i) F(*)
iEG *:X	j) (E X) F(X)

16) Existential specification

	i)	(E X) F(X,Y)
iES X:*Y	j)	F(*Y, Y)

17) Form a conjunction

	i)	R
	j)	Q
i.jFC	k)	R & Q

18) Form a disjunction

	i)	R
iFD ::Q	j)	R OR Q

19) Hypothetical syllogism

	i)	P $\rightarrow$ R
	j)	R $\rightarrow$ Q
i.jHS	k)	P $\rightarrow$ Q

20) Indirect proof procedure

WP	i)	R	line i must be
	j)	Q	the last working premise
	k)	NOT Q	introduced
i.j.kIP	m)	NOT R	

21) Law of the biconditional

	i)	R IFF Q	i) (R $\rightarrow$ Q) & (Q $\rightarrow$ R)
iLB	j)	(R $\rightarrow$ Q) & (Q $\rightarrow$ R)	j) R IFF Q

22) Left conjunct

	i)	R & P
iLC	j)	R

23) Logical truth

LT ::X	i)	X = X

24) Quantifier negation rule A

```
        i) (A X) S(X)                  i) NOT (E X) NOT S(X)
iQNA    j) NOT (E X) NOT S(X)          j) (A X) S(X)
```

25) Quantifier negation rule B

```
        i) (A X) NOT S(X)              i) NOT (E X) S(X)
iQNB    j) NOT (E X) S(X)              j) (A X) NOT S(X)
```

26) Quantifier negation rule C

```
        i) (E X) S(X)                  i) NOT (A X) NOT S(X)
iQNC    j) NOT (Z X) NOT S(X)          j) (E X) S(X)
```

27) Quantifier negation rule D

```
        i) (E X) NOT S(X)              i) NOT (A X) S(X)
iQND    j) NOT (A X) S(X)              j) (E X) NOT S(X)
```

28) Right conjunct

```
        i) R & P
iRC     j) P
```

29) Replace equals

```
        i) B+C = B+C
        j) B+C = 3
i,jRE   k) B+C = 3
```

30) Universal generalization

```
        i) F(X)
iUG
:X      j) (A X) F(X)
```

31) Universal specification

```
        i) (A X) F(X)
iUS
X:Y     j) F(Y)
```

32) Working premise

```
WP      i)    R
```

Other Commands

1) COPY a line

	i) R	(useful for checking on what the computer
iCOPY	j) R	thinks you typed)

2) Delete lines

iDLL deletes all lines beginning with line i

3) FIN logs you off the computer

4) Hypothesis

HYP creates a working premise for you

If the statement of the problem is	HYP gives
R → Q	R
NOT R	R
R	NOT R

5) Initiative (is available at most points at which you are to type a response)

INIT lets you ask for your own problems; program will always return to the problem you interrupted

you can request

DERIVE	(request a derive problem--you may use P, for PREMISE, as the initial commands)
FA	(request finding-axioms exercise)
LESSON	(request a different lesson and problem)
NEWS	(request the news of the day, i.e., computer schedule, program changes, class meetings; also available when you are constructing a derivation)
PROVE	(when a PROVE problem is completed, you can name the expression as a theorem)

6) Review the derivation.

REVIEW the computer will type each command and proof line, any flagged variables, and the premise lines on which the flagging depends

7) Requests for problem types when you are given the choice

CEX	counterexample problem counterexample by assignment of truth values
DER	derivation problem
INT	interpretation problem

8) RESTART (available only when doing interpretation problems)

will let you change your interpretation

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